TRIANGLES – MODULE 3/5

Criteria for Similarity of Triangles:

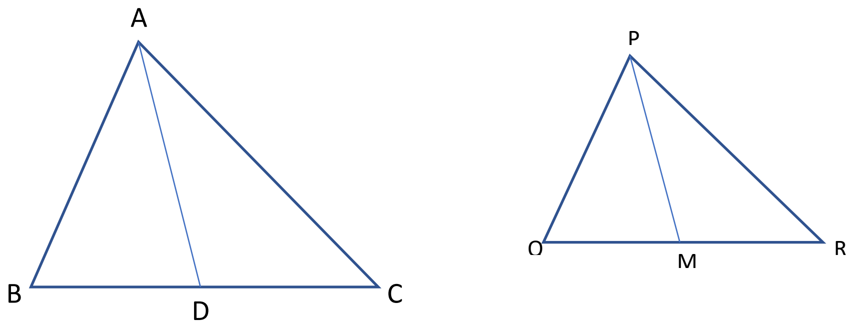
1. ***Angle – Angle – Angle Similarity Criterion***
   If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar

   Corollary: Angle – Angle Similarity Criterion

   If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

2. ***Side – Side – Side Similarity Criterion:***
   If in two triangles, sides of one triangle are proportional to (i.e., in the same ratio of) the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar.

Solved Example based on SSS similarity criterion:
Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of Δ PQR. Show that $\Delta\ ABC \sim \Delta\ PQR$.



Solution:

Given:

1) $AB/PQ = BC/QR = AD/PM$
2) AD and PM are the medians of the triangles ABC and PQR

To prove: $\Delta\ ABC \sim \Delta\ PQR$

Proof:

Since AD and PM are the medians of Δ ABC and Δ PQR, BD = BC/2 and QM = QR/2

Also it is given that $AB/PQ = BC/QR = AD/PM$

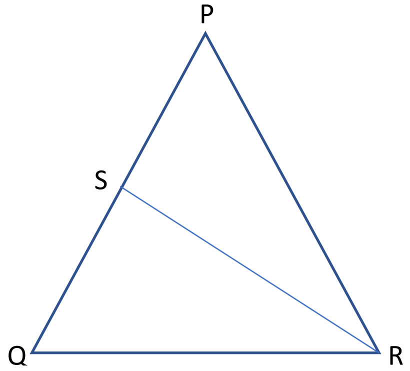
$$\Longrightarrow \frac{AB}{PQ} = \frac{2BD}{2QM} = \frac{AD}{PM}$$

$$\Longrightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

Therefore by SSS congruence condition, $\Delta\ ABC \sim \Delta\ PQR$

3. ***Side – Angle – Side Similarity Criterion:***
If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

Solved Example based on SAS similarity criterion:
The given figure shows an isosceles triangles PGR in which PQ = PR. S is a point on PQ. Also, QR^2 = PQ x QS, and SR = 2.4 cm. What is the length of QR?



Solution:

It is given that QR^2 = PQ x QS. Therefore, QR^2 = PR x QS (As PQ = PR)

$\Longrightarrow \frac{QR}{PR} = \frac{QS}{QR}$

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Also, $\angle R = \angle Q$ (As PQ = PR)

Therefore, $\Delta QRP \sim \Delta SQR$ (by SAS similarity)

$\Longrightarrow \frac{QR}{QS} = \frac{PR}{QR}$ (corresponding sides of similar triangles)

$\Longrightarrow \frac{PR}{QR} = \frac{PQ}{SR} \Longrightarrow \frac{1}{QR} = \frac{1}{SR}$ (As PQ = PR)

$\Longrightarrow$ QR = SR

It is given that SR = 2.4 cm. Thus, the length of QR is also 2.4 cm