Class XII Mathematics
Chapter 4- Determinants
<u>Hand out of Module-4/4</u>

TOPIC : APPLICATION OF DETERMINANTS & MATRICES

<u>Previous Knowledge</u>:-

- Finding the value of a determinant.
- Finding the Area of a triangle whose vertices are(x_1 , y_1), (x_2, y_2) & (x_3, y_3).
- Collinear points.
- Non singular matrices.
- Finding the inverse of a matrix.
- Consistent and inconsistent system of equations

1. **<u>Area of a Triangle</u>**

In earlier classes we have studied that the area of a triangle whose vertices are (x_1, y_1), (x_2, y_2) & (x_3, y_3) is given by

$$\Delta= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[x_1(y_2 - y_3) - x_2(y_1 - y_3) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Note:-

1) Area is a positive quantity. Therefore, take the absolute value the determinant.
2) If area is given, use both positive and negative values of the determinant for calculation.
3) Area of the triangle formed by three collinear points is zero.

Example 1:- Find the area of the triangle whose vertices are (1, 2), (3, 4) and (-2, 1).

Solution ; Area of triangle is given by

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 3 & -4 & 1 \\ -2 & 1 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[1(-4-1) - 2\,(3+2) + \; 1(3-8\,)]$$

$$= \frac{1}{2}[-5-10-5] = \; -10$$

Therefore area of triangle = 10 sq. units

Example 2:- Find x if area of the triangle formed by the points A(x,3) , B(3,2) and C(6,4) is 9 square units

Solution:- Area of triangle ABC= $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} x & 3 & 1 \\ 3 & 2 & 1 \\ 6 & 4 & 1 \end{vmatrix}$

Therefore, $\frac{1}{2}\begin{vmatrix} x & 3 & 1 \\ 3 & 2 & 1 \\ 6 & 4 & 1 \end{vmatrix} = \pm 9$

That is , -2x +9 = $\pm$18

Which gives x = $\frac{-9}{2}$ or x = $\frac{27}{2}$

Example 3:- Find equation of the line joining the points (2, 4) and (3, 6) using determinants.

Solution : Let P(x,y) be any point on the line joining A(2, 4) and B(3, 6)

Then the points P, A and B are collinear.

Therefore area of $\Delta PAB = 0$

This gives $\frac{1}{2}\begin{vmatrix} x & y & 1 \\ 2 & 4 & 1 \\ 3 & 6 & 1 \end{vmatrix} = 0$

Or x(4 – 6) –y (2-3) +1(12-12) = 0

2x – y = 0

Therefore, equation of the line joining the points (2, 4) and (3, 6) is 2x - y = 0

2. Solution of a system of linear equations using inverse of a matrix

Consider the system of equations

$a_1x + b_1y + c_1z = d_1$

$a_2x + b_2y + c_2z = d_2$

$a_3x + b_3y + c_3z = d_3$

Let A= $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$, X= $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B= $\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$

Then, the system of equations can be written as, AX = B,

i.e. $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$

Now, AX = B

pre multiplying by A^{-1} we get, $A^{-1}(AX) = A^{-1}B$

or$(A^{-1}A)\, X = A^{-1}B$ (by associative property)

or $I\, X = A^{-1}B$

or $X = A^{-1}B = \dfrac{\text{adj}A\,.B}{|A|}$

Case I **:** If $|A| \neq 0$, then A^{-1} exists and is unique. Then the system of equations will have a unique solution. [This method of solving is known as Matrix Method.]

Case 2: If $|A| = 0$ and (adj A) B $\neq 0$ then solution does not exist and the system of equations is called inconsistent.

C:ase3: If $|A| = 0$ and (adj A) B = O, then system may be either consistent or inconsistent according as the system have either infinitely many solutions or no solution.

Example 4: Examine the consistency of the system of equations

$5x – y + 4z = 5$

$2x + 3y + 5z = 2$

$5x – 2y + 6z = –1$

Solution : -

These equations can be written as

$$\begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$$

Or AX = B where A = $\begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B = $\begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$

$|A| = 5\,(18+ 10) +1\,(12 - 25) + 4\,(-4 - 15) = 140 -13 - 76 = 51 \neq 0$

Therefore the system of equations is consistent.

Example 5 :

The sum of three numbers is 2. If twice the second number is added to the sum of first and third, the sum is 1. By adding second and third number to five times the first number, we get 6. Find the numbers by using matrices.

Solution:- Let the numbers be x, y & z

Then, from the first condition we get, $x + y + z = 2$

from the second condition we get, $x +2 y + z = 1$

from the third condition we get, $5x + y + z = 6$

These equations can be written as

$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 5 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix}$ or AX = B where, A = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 5 & 1 & 1 \end{bmatrix}$, X= $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B = $\begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix}$

adj.A = $\begin{bmatrix} 1 & 0 & -1 \\ 4 & -4 & 0 \\ -9 & 4 & 1 \end{bmatrix}$, $|A| = 1(1) - 1(-4) + 1(-9) = 1 + 4 - 9 = -4$

$$A^{-1} = \frac{\text{adjA}}{|A|} = \frac{1}{-4}\begin{bmatrix} 1 & 0 & -1 \\ 4 & -4 & 0 \\ -9 & 4 & 1 \end{bmatrix}$$

$X = A^{-1}B$ or $X = \frac{1}{-4}\begin{bmatrix} 1 & 0 & -1 \\ 4 & -4 & 0 \\ -9 & 4 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix}$

$$= \frac{1}{-4}\begin{bmatrix} 2+0-6 \\ 8-4+0 \\ -18+4+6 \end{bmatrix}$$

$$= \frac{1}{-4}\begin{bmatrix} -4 \\ 4 \\ -8 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ OR x = 1, y = -1 and z = 2

Therefore the numbers are 1, -1, 2

SUMMARY

- To every square matrix A= $[a_{ij}]$ of order n, we can associate a number (real or complex) called determinant of matrix A and is denoted by det A or |A|.
- PROPERTIES OF DETERMINANTS

  1)The value of the determinant remains unchanged if its rows and columns are interchanged. That is, if A is any square matrix, then |A| = | A′|

  2) If any two rows (or columns) of a determinant are interchanged, then sign of determinant changes.

  3) If corresponding elements of any two rows (or columns) of a determinant are proportional (in the same ratio), then its value is zero

  4) If each element of a row (or a column) of a determinant is multiplied by a constant k, then its value gets multiplied by k.

  5) If some or all elements of a row or column of a determinant are expressed as sum of two (or more) terms, then the determinant can be expressed as sum of two (or more) determinants

  6) If, to each element of any row or column of a determinant, the equimultiples of corresponding elements of other row (or column) are added, then value of determinant remains the same, i.e, the value of determinant remain same if we apply the operation $R_i \to R_i$ + kR_j or $C_i \to C_i$ + kC_j

- Minor and Cofactor of an element.

- Value of a determinant is the sum of product of elements of a row (or a column) with corresponding cofactors.
- Adjoint of a square matrix A = $[Aij]'$, where Aij is the cofactor of a_{ij}.
- If A is a square matrix of order n, then A(adj A) = (adj A) A =|A| I
- If A is a square matrix of order n, then |A.adj(A)| = $|A|^n$
- A square matrix A is said to be non-singular if |A| $\neq 0$
- If A and B are square matrices of the same order, then |AB| = |A| |B|
- If A is a square matrix of order n, then |adjA| = $|A|^{n-1}$
- A square matrix A is invertible (i. e A^{-1} exists) if |A|$\neq 0$ and $A^{-1} = \frac{(\text{adj } A)}{|A|}$.
- Solution of the system of equations represented as AX = B is given by

  X = A^{-1} B

  Case I : If A|$\neq$ 0, then the system of equations will have a unique solution.

  Case 2: If |A| = 0 and (adj A) B $\neq$ 0 then solution does not exist and the system of equations is called inconsistent.

  Case3: |A| = 0 and (adj A) B = O, then system have either infinitely many solutions or no solution.

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