Class-X MATHEMATICS

CHAPTER-12

AREAS RELATED TO CIRCLES

MODULE: 2/2

AREAS OF COMBINATIONS OF PLANE FIGURES

-We learned in the previous module that the circumference and area of a circle , the length of an arc of the sector of a circle, areas of sector and segment of a circle.

-So far we have calculated the areas of different plane figures separately . In this module let us try to calculate the areas of some combinations of plane figures .

Some designs of combinations of plane figures.

We come across such type of figures in our day to day life and also in the form of very interesting designs like flower beds, drain covers, window designs, designs on table covers are some of such examples.

Window design

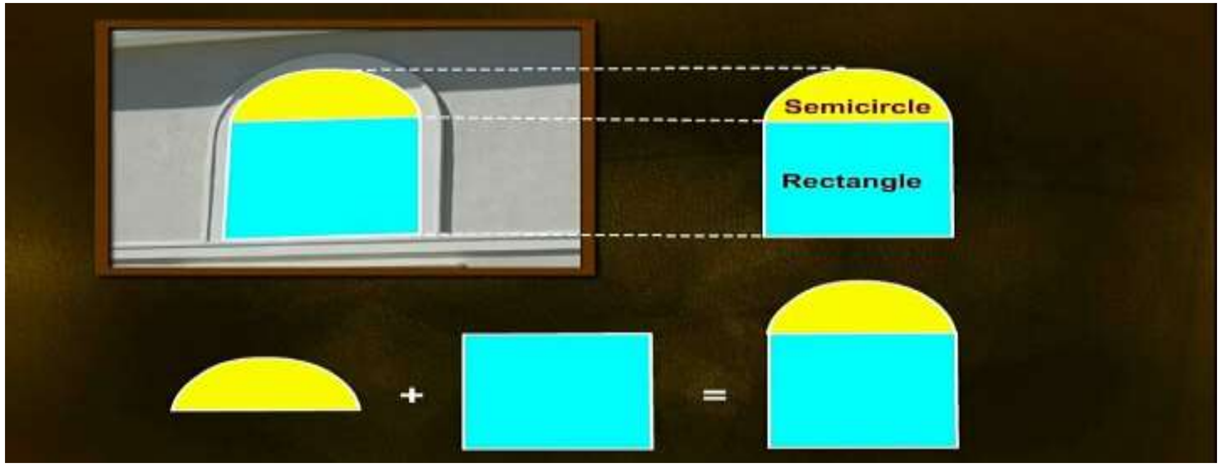
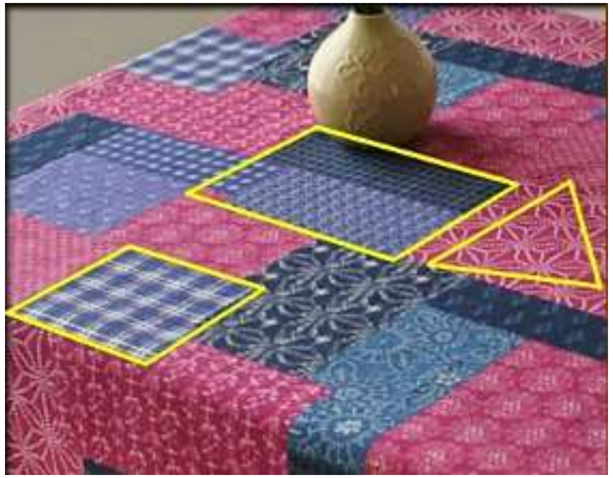


Table cloth



Example 1: Find the area of the shaded region in this figure, if PQ = 24cm, PR= 7cm and O is the centre of the circle.

Solution: Here RQ is a diameter of the circle,

So PQR is a right angled triangle,

Hence, <RPQ= 90^0,

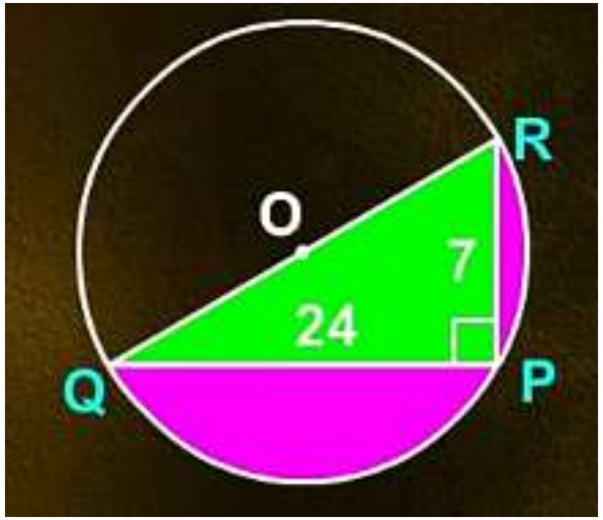
By Pythagoras theorem

In triangle PQR, $RP^2+PQ^2= RQ^2$

Or, $(7)^2+(24)^2=RQ^2$

Or, RQ= $\sqrt{49+576} = \sqrt{625}$= 25cm

Therefore, radius OR= 25/2 cm



Area of semicircle RPQOR = $\frac{1}{2}\pi r^2$

$\frac{1}{2}$ x $\frac{22}{7}$x$(\frac{25}{2})^2$

= 6875/28 sq.cm

Area of triangle PQR= $\frac{1}{2}$x PQ x PR= $\frac{1}{2}$x 24 x 7 = 84 sq.cm

Area of shaded region = area of semicircle RPQOR – area of ΔPQR = $\frac{6875}{28} - 84 = 161.53$ sq. cm

Example 2: Find the area of the shaded region, if radii of two concentric circles with centre O are 7cm and 14cm respectively and <AOC= 40^0

Solution: Radius of inner circle= 7cm

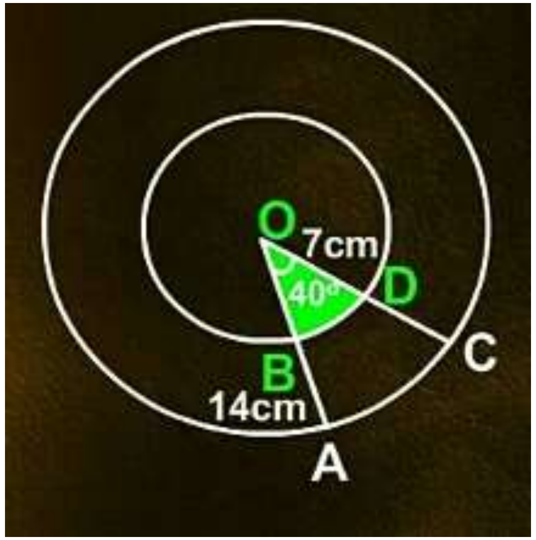
Radius of outer circle=14cm

<AOC= 40^0

Area of sector = $\frac{\theta}{360^0}$ x πr^2

Area of sector OBD = $\frac{40^0}{360^0}$ x $\frac{22}{7}$ x 7^2 = 154/9 = 17.11 sq.cm

Area of sector OAC =$\frac{40^0}{360^0}$ x $\frac{22}{7}$ x 14^2 = 616/9= 68.44 sq.cm



Area of shaded region = area of sector OAC – area of sector OBD

= (68.44 – 17.11)sq.cm

= 51.33 sq.cm

Example 3: In a circular table cover of radius 32cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in the Fig. Find the area of the design.

Solution: Area of circle= $\pi r^2 = \frac{22}{7}$ x 32 x 32

= 22528/7

= 3218.28 sq.cm

Radius AO = 32 cm

AO= $\frac{2}{3}$AD [AO:OD=2:1]

Therefore, AD = $\frac{3}{2}$ x 32 = 48 cm

By Pythagoras theorem , $AB^2 = AD^2 + BD^2$

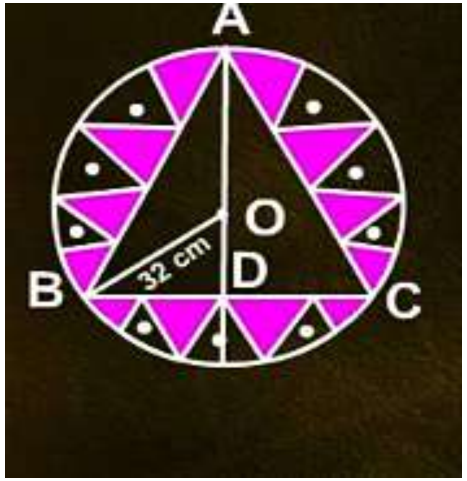
or, $AB^2 = 48^2 + (\frac{AB}{2})^2$

or, AB = 48 x $\frac{2}{\sqrt{3}}$ = 32 $\sqrt{3}$

∴ Area of $\Delta\, ABC$ = $\frac{\sqrt{3}}{4}$ AB^2

= $\frac{\sqrt{3}}{4}$ x $32\sqrt{3}$ x $32\sqrt{3}$ = $768\sqrt{3}$ sq.cm

Area of design = area of circle – area of triangle ABC = 3218.28 – 1330.2 = 1888.07 sq.cm



Example 4: On a square handkerchief , 9 circular designs each of radius 7 cm are made. Find the area of the remaining portion of the handkerchief.

Solution: Area of a circle= πr^2 = $\frac{22}{7}$ x 7 x 7= 154 sq.cm

∴ area of 9 circles = 9 x 154 sq.cm

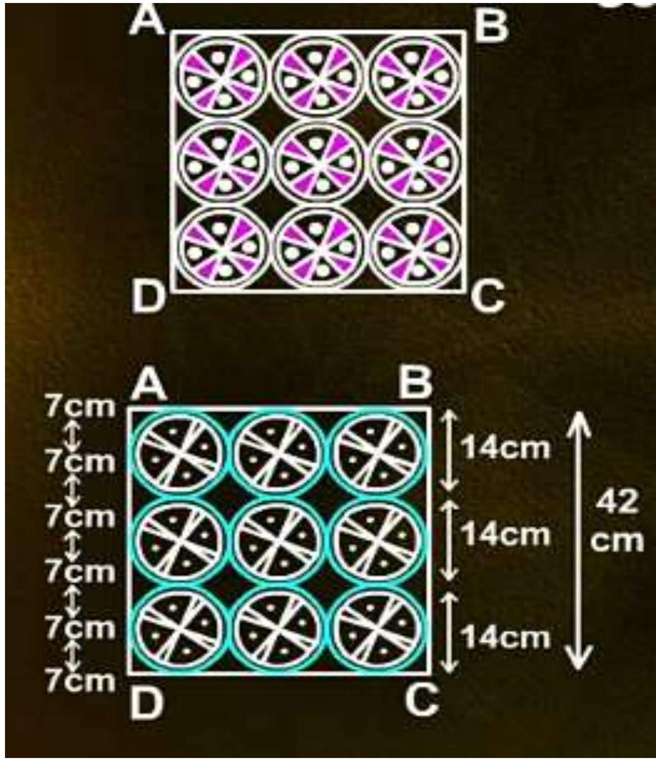
=1386 sq.cm

Now, area of remaining portion of handkerchief

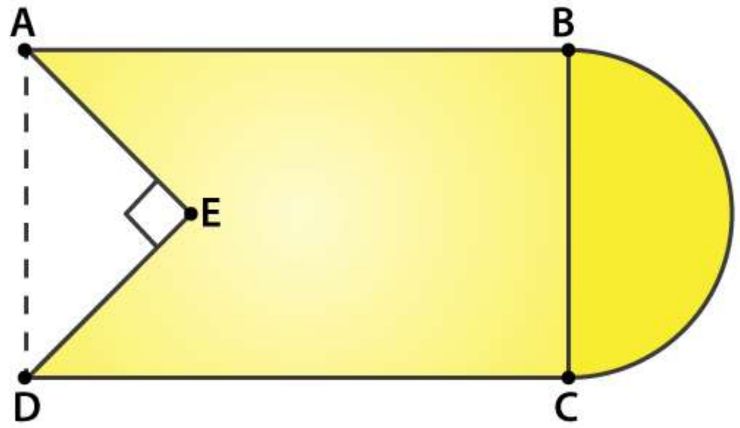
= area of square – area of 9 circles

= 42 x 42 -1386

=1764 -1386 = 378 sq.cm



Example 5: From a rectangular region ABCD with AB = 20 cm, a right angle AED with AE = 9 cm and DE = 12 cm, is cut off. On the other end, taking BC as diameter, a semicircle is added on outside the region. Find the area of the shaded region. ($\pi = 22/7$)



Given,

Length of the rectangle ABCD = 20 cm

AE = 9 cm and DE = 12 cm

The radius of the semi-circle = BC/ 2 or AD/2

Now, using Pythagoras theorem in triangle AED

AD = $\sqrt{(AE^2 + ED^2)} = \sqrt{(9^2 + 12^2)}$

$= \sqrt{(81 + 144)}$

$= \sqrt{(225)} = 15$ cm

So, the area of the rectangle = 20 x 15 = 300 cm^2

And, the area of the triangle AED = ½ x 12 x 9 = 54 cm^2

The radius of the semi-circle = 15/2 = 7.5 cm

Area of semi-circle = $\frac{1}{2}\pi(15/2)^2 = \frac{1}{2} \times 3.14 \times 7.5^2 = 88.31$ cm^2

Thus,

The area of the shaded region = Area of the rectangle ABCD + Area of semi-circle – Area of triangle AED

= 300 + 88.31 – 54 = 334.31 cm^2

Thank You!