Atomic Energy Education Society, Mumbai

Class XI Subject-Physics

Chapter–4: Motion in a Plane

Module 1/2

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CONTENTS

- **Scalar and vector quantities;**
- **position and displacement vectors,**
- **general vectors and their notations;**
- **equality of vectors,**
- **multiplication of vectors by a real number;**
- **addition and subtraction of vectors,**
- **relative velocity,**
- **Unit vector;**
- **resolution of a vector in a plane,**
- **rectangular components,**
- **Scalar and Vector product of vectors.**

SCALARS :

- A scalar quantity is a quantity with magnitude only.
- It is specified completely by a single number, along with the proper unit.

Examples

- The distance between two points,
- Mass of an object,
- The temperature of a body and
- The time at which a certain event happened.
- **The rules for combining scalars:** are the rules of ordinary algebra.

Scalars can be added, subtracted, multiplied and divided just as the ordinary numbers

Vector Quantity :

- It is a quantity that has both a magnitude and a direction
- It obeys the triangle law of addition or equivalently the parallelogram law of addition.
- So, a vector is specified by giving its magnitude by a number and its direction.

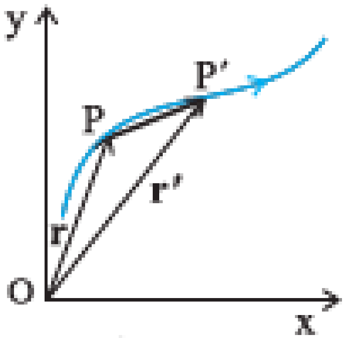
Examples

- Displacement, Velocity, Acceleration and Force etc.

Representation of Vectors

- To represent a vector, we use a bold face type in some books.
- Thus, a velocity vector can be represented by a symbol **v.**
- Since bold face is difficult to produce, when written by hand, a vector is often
- represented by an arrow placed over a letter, say $\vec{v}$.
- Thus, both **v** and $\vec{v}$ represent the velocity vector.
- The magnitude of a vector is often called its absolute value, indicated by |**v**| = *v*

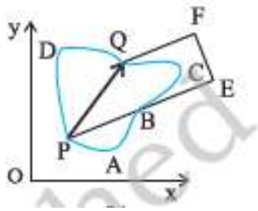
Position and Displacement Vectors:



- Let us consider O as origin.
- Let P and P′ be the positions of the object at time *t* and *t*′, respectively.
- We join O and P by a straight line. Then, **OP** is the position vector of the object at time *t*.
- An arrow is marked at the head of this line. It is represented by a symbol **r**, i.e. **OP** = **r**.
- Point P′ is represented by another position vector, **OP′** denoted by **r** ′.
- The length of the vector **r** represents the magnitude of the vector and its direction is the direction in which P lies as seen from O.
- If the object moves from P to P′, the vector **PP′** (with tail at P and tip at P′) is called the **displacement vector** corresponding to motion from point P (at time *t*) to point P′ (at time *t*′).
- Displacement vector is the straight line joining the initial and final positions

Displacement vector is independent on the actual path:

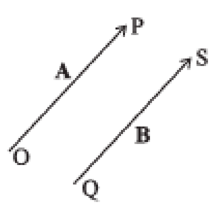
- Displacement vector does not depend on the actual path undertaken by the object between the two positions.



- In the given figure, the initial and final positions as P and Q, the displacement vector is the same **PQ** for different paths of journey, say PABCQ, PDQ, and PBEFQ.
- Therefore, the magnitude of displacement is either less or equal to the path length of an object between two points.

Equality of Vectors:

Two vectors **A** and **B** are said to be equal if, and only if, they have the same magnitude and the same direction.



- In general, equality is indicated as **A = B**.

MULTIPLICATION OF VECTORS BY REAL NUMBERS:

(i) Multiplying a vector A with a positive number:

Multiplying a vector **A** with a positive number n gives a vector whose magnitude is changed by λ the factor but the direction is the same as that of **A** :

$$|\lambda \mathbf{A}| = \lambda |\mathbf{A}| \text{ if } \lambda > 0.$$

For example

- If **A** is multiplied by 2, the resultant vector 2**A** is in the same direction as **A** and has a magnitude twice of **|A|** as shown in the given figure

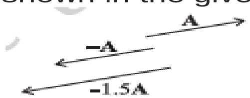
(ii) **Multiplying a vector A by a negative number:**

- Multiplying a vector **A** by a negative number λ gives a vector λ**A** whose direction is opposite to the direction of **A** and whose magnitude is λ times **|A|**.

.

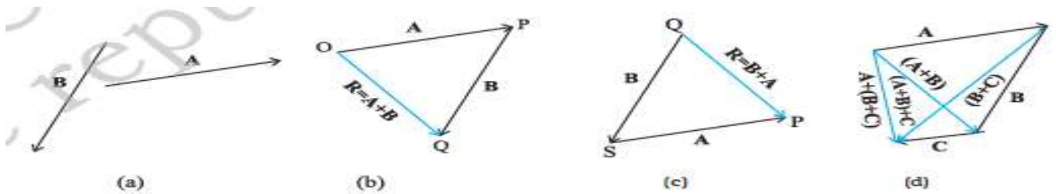
For example

- Multiplying a given vector **A** by negative numbers, say -1 and -1.5 gives vectors as shown in the given figure.



ADDITION OF VECTORS -GRAPHICAL METHOD

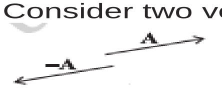
- We shall now describe this law of addition using the graphical method.
- Let us consider two vectors **A** and **B** that lie in a plane as shown in the figure (a)



- The lengths of the line segments representing these vectors are proportional to the magnitude of the vectors.

To Find The Sum of A + B By Head-To-Tail Method:

- We place vector **B** so that its tail is at the head of the vector **A**, as in Fig. (b).
- Then, we join the tail of **A** to the head of **B**.
- This line OQ represents a vector **R,**
- That is, the sum of the vectors **A** and **B**.
- Since, in this procedure of vector addition, vectors are arranged head to tail, this graphical method is called the **head-to-tail method**.
- The two vectors and their resultant form three sides of a triangle, so this method is also known as **triangle method of vector addition**.
- If we find the resultant of **B** + **A** as in Fig.(c), the same vector **R** is obtained. Thus, vector addition is **commutative**: **A** + **B** = **B** + **A**
- The addition of vectors also obeys the **associative law as** illustrated in Fig.(d).
- The result of adding vectors **A** and **B** first and then adding vector **C** is the same as the result of adding **B** and **C** first and then adding vector **A** :
- (**A** + **B**) + **C** = **A** + (**B** + **C**)
- **Null Vector Or A Zero Vector :**
- Consider two vectors **A** and **A** shown in below figure.



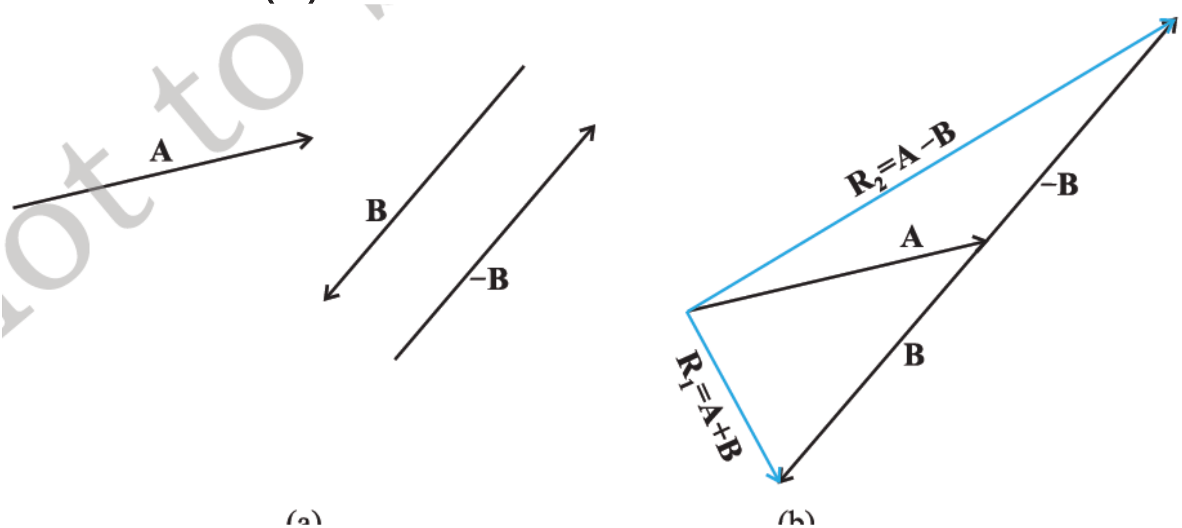
- Their sum is **A** + (- **A**). Since the magnitudes of the two vectors are the same, but the directions are opposite,
- The resultant vector has zero magnitude and is represented by **0** called a **null vector** or a **zero vector** :
- **A** - **A** = **0** |**0**|= 0

Physical Meaning Of A Zero Vector:

- When the initial and final positions coincide, the displacement is a .null vector..

Subtraction of vectors:

- We define the difference of two vectors **A** and **B** as the sum of two vectors **A** and **- B**
- **A - B = A + (-B)**

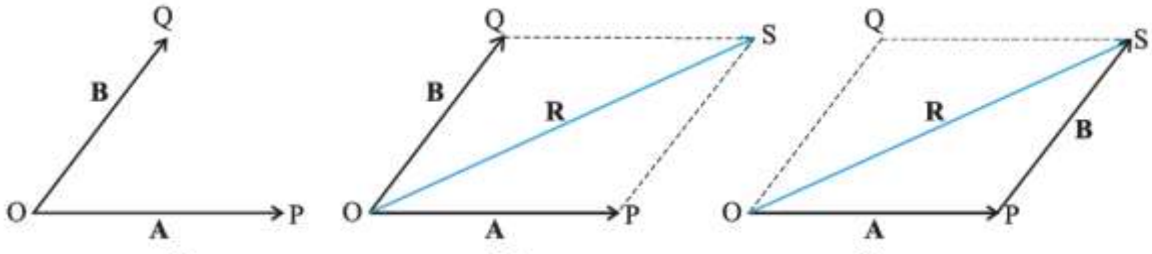


It is shown in above figures .

- The vector **- B** is added to vector **A** to get $\mathbf{R}_2$ = (**A - B**).

The vector $\mathbf{R}_1$ = **A** + **B** is also shown in the same figure for comparison

Parallelogram Method To Find The Sum Of Two Vectors:



(a) (b) (c)

Suppose we have two vectors **A** and **B**. To add these vectors, we bring their tails to a common origin O as shown in Fig. (a).

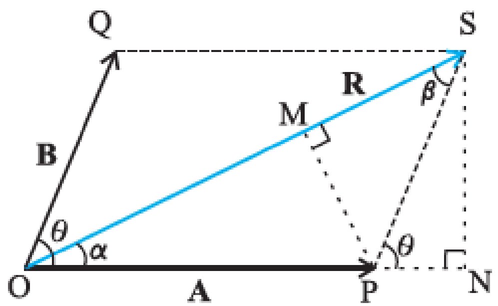
- Then we draw a line from the head of **A** parallel to **B** and another line from

the head of **B** parallel to **A** to complete a parallelogram OQSP.

- Now we join the point of the intersection of these two lines to the origin O.
- The resultant vector **R** is directed from the common origin O along the diagonal (OS) of the parallelogram [Fig.(b)].
- In Fig.(c), the triangle law is used to obtain the resultant of **A** and **B**
- And we see that the two methods yield the same result.
- Thus, the two methods are equivalent.
- Let **OP** and **OQ** represent the two vectors
- **A** and **B** making an angle θ (Fig. 4.10). Then,
- using the parallelogram method of vector
- addition, **OS** represents the resultant vector **R** :
- **R** = **A** + **B**

Law Of Parallelogram For Vector Addition:

It state that if two vectors are represented by two adjacent sides of a parallelogram then magnitude and direction of resultant vector is given by its intersection diagonal



Let **OP** and **OQ** represent the two vectors **A** and **B** making an angle θ (in the given Fig.) Then, using the parallelogram method of vector addition, **OS** represents the resultant vector **R** :

$\mathbf{R} = \mathbf{A} + \mathbf{B}$

SN is normal to *OP* and *PM* is normal to *OS*. From the geometry of the figure,

$OS^2 = ON^2 + SN^2$

but $ON = OP + PN = A + B \cos\theta$ and $SN = B \sin\theta$

$OS^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$

or, $R^2 = A^2 + B^2 + 2AB\cos\theta$

$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$ This is known as the **Law of cosines**

In Δ OSN, $SN = OS \sin\alpha = R \sin\alpha$, and

in Δ PSN, $SN = PS \sin\theta = B \sin\theta$

Therefore, $R \sin\alpha = B \sin\theta$

$$\text{or,} \quad \frac{R}{\sin\theta} = \frac{B}{\sin\alpha}$$

Similarly,

PM = *A* sin α = *B* sin β

$$\text{or,} \quad \frac{A}{\sin\beta} = \frac{B}{\sin\alpha}$$

Combining above Eqs. we get

$$\frac{R}{\sin\theta} = \frac{A}{\sin\beta} = \frac{B}{\sin\alpha}$$

- This is known as the **Law of sines**.

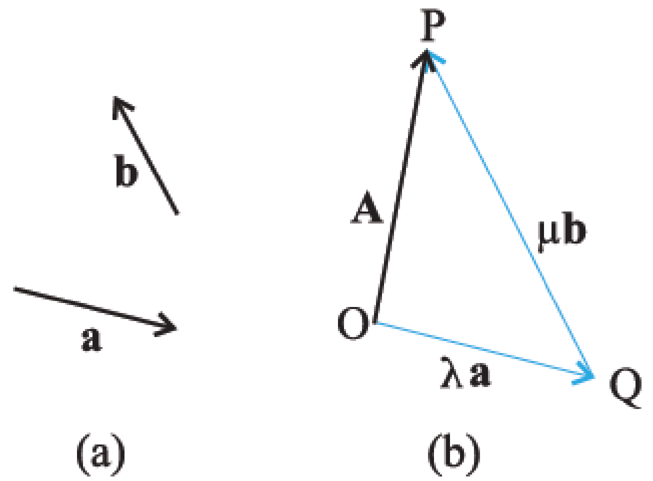
$$\tan\alpha = \frac{SN}{ON}$$

$$\tan\alpha = \frac{B\ \sin\theta}{A + B\cos\theta}$$

This equation gives the direction of resultant vector.

RESOLUTION OF VECTORS:

- Let **a** and **b** be any two non-zero vectors in a plane with different directions and let **A** be another vector in the same plane in below Fig.



- **Fig** (a) *Two non-colinear vectors* **a** *and* **b***.* (b) *Resolving a vector* **A** *in terms of vectors* **a** *and* **b**.

- **A** can be expressed as a sum of two vectors .

- One obtained by multiplying **a** by a real number and

- the other obtained by multiplying **b** by another real number.

- To see this, let O and P be the tail and head of the vector **A.**

- Then, through O, draw a straight line parallel to **a**, and through P, a straight line parallel to **b**.
- Let them intersect at Q. Then, we have

- **A** = **OP** = **OQ** + **QP**

- But since **OQ** is parallel to **a**, and **QP** is parallel to **b**,

- we can write : **OQ** = λ **a**, and **QP** = *μ* **b** where λ and *μ* are real numbers.

- Therefore, **A** = λ **a** + *μ* **b**
- We say that **A** has been resolved into two component vectors λ **a** and μ **b** along **a** and **b** respectively.
- Using this method one can resolve a given vector into two component vectors along a set of two vectors .
- all the three lie in the same plane.
- It is convenient to resolve a general vector along the axes of a rectangular coordinate system using vectors of unit magnitude. These are called **unit vectors ..**

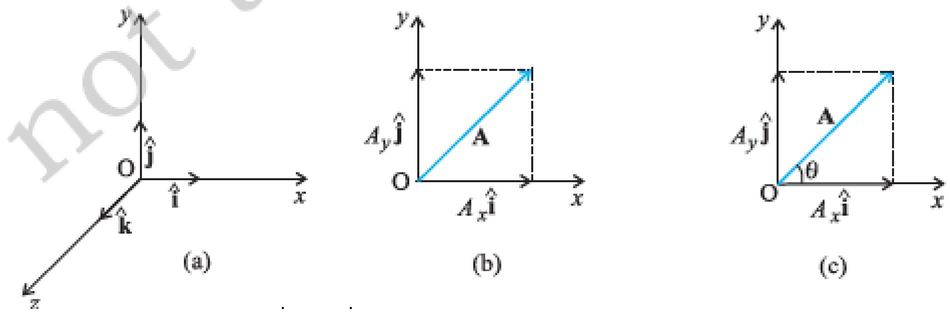
Unit vectors: A unit vector is a vector of unit magnitude and points in a particular direction.

$$\hat{r} = \frac{\vec{r}}{|r|}$$

- It has no dimension and unit.
- It is used to specify a direction only.

Rectangular Unit Vectors:

- Unit vectors along the *x*-, *y* and *z*-axes of a rectangular coordinate system are denoted by $\hat{i}$, $\hat{j}$ and $\hat{k}$ respectively, as shown in Fig. (a).



- Since these are unit vectors, we have

$$|\hat{\mathbf{i}}| = |\hat{\mathbf{j}}| = |\hat{\mathbf{k}}| = 1$$

- These unit vectors are perpendicular to each other.

- In this text, they are printed in bold face with a cap (^) to distinguish them from other vectors.

- Since we are dealing with motion in two dimensions in this chapter, we require use of only two unit vectors.

- If we multiply a unit vector, say **n** by a scalar, the result is a vector λ = λ **n** .

- In general, a vector **A** can be written as **A** = |**A**| **n**

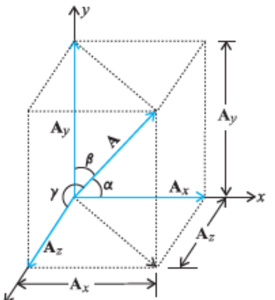
- where **n** is a unit vector along **A**.

- We can now resolve a vector **A** in terms of component vectors that lie along unit vectors **i** and "**j**.
- Consider a vector **A** that lies in *x*-*y* plane as shown in Fig. (b).
- We draw lines from the head of **A** perpendicular to the coordinate axes as in Fig. (b), and get vectors $\mathbf{A_1}$ and $\mathbf{A_2}$ such that $\mathbf{A_1} + \mathbf{A_2} = \mathbf{A}$.
- Since **A1** is parallel to ‘i and **A2** is parallel to "**j**,
- we have : **A1**= *Ax* "**i** , **A2** = *Ay* "**j** where *Ax* and *Ay* are real numbers.
- Thus, **A** = *Ax* "**i** + *Ay* "**j**
- This is represented in Fig.(c).
- The quantities *Ax* and *Ay* are called *x*-, and *y*- components of the vector **A**.
- Note that *Ax* is itself not a vector, but *Ax* "**i** is a vector, and so is *Ay*"**j**.
- Using simple trigonometry, we can express *Ax* and *Ay* in terms of the magnitude of **A** and the angle θ it makes with the *x*-axis :
- $A_x = A \cos\theta$
- $A_y = A \sin\theta$

Resolution Vector Into Three Dimensions.

- The same procedure can be used to resolve a general vector **A** into three components along *x*-, *y*-, and *z*-axes in three dimensions.

- If α, β, and γ are the angles between **A** and the *x*-, *y*-, and *z*-axes, respectively



(d)

- From Fig. (d), we have

- $A_x = A\cos\alpha$, $A_y = A\cos\beta$, $A_z = A\cos\gamma$

- In general, we have $A = A_x\,i + A_y\,j + A_z\,k$

- The magnitude of vector **A** is

$$A = \left[A_x^2 + A_y^2 + A_z^2\right]^{1/2}$$

- A position vector **r** can be expressed as

$$\mathbf{r} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$$

where *x, y*, and *z* are the components of **r** along *x-, y-, z*-axes, respectively.

RELATIVE VELOCITY IN TWO DIMENSIONS:

- The concept of relative velocity, for motion along a straight line, can be easily extended to include motion in a plane or in three dimensions.
- Suppose that two objects A and B are moving with velocities $\mathbf{v}_A$ and $\mathbf{v}_B$ (each with respect to some common frame of reference, say ground.).
- Then, velocity of object A relative to that of B is :

$$\mathbf{v}_{AB} = \mathbf{v}_A - \mathbf{v}_B$$

- and similarly, the velocity of object B *relative to that of* A is :

$$\mathbf{v}_{BA} = \mathbf{v}_B - \mathbf{v}_A$$

Therfore

$$\mathbf{v}_{AB} = - \mathbf{v}_{BA}$$

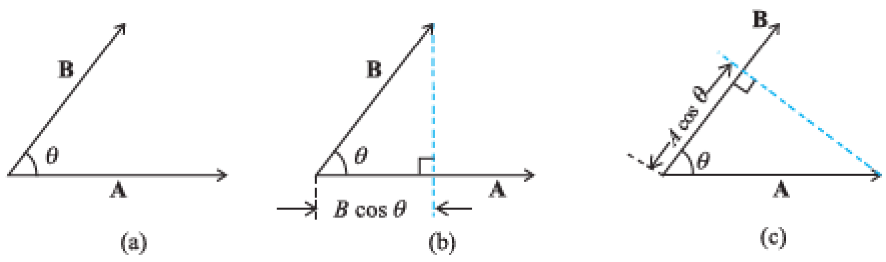
and $\lfloor \mathbf{V}_{AB} \rfloor = |\boldsymbol{V}_{BA}|$

The Scalar Product:

- The scalar product or dot product of any two vectors **A** and **B**, denoted as **A . B** (read **A** dot **B**) is defined as
- **A.B** = A B cos θ where θ is the angle between the two vectors as shown in Fig. (a).
- Since A, B and cos θ are scalars, the dot product of **A** and **B** is a scalar quantity. Each vector, **A** and **B**, has a direction but their scalar product does not have a direction.

- we have **A.B** = A (B cos θ) = B (A cos θ)
- Geometrically, B cos θ is the projection of **B** onto **A** in Fig. (b)
- and A cos θ is the projection of **A** onto **B** in Fig. (c).

- So, **A.B** is the product of the magnitude of **A** and the component of **B** along **A**.
- Alternatively, it is the product of the magnitude of **B** and the component of **A** along **B**



Properties of scalar product:

- The scalar product follows the commutative law : **A.B** = **B.A**

- Scalar product obeys the **distributive law**: **A.** (**B** + **C**) = **A.B** + **A.C**

- Further, **A.** (λ **B**) = λ (**A.B**) where λ is a real number.

- For unit vectors I, j,and k we have

For unit vectors $\hat{i}, \hat{j}, \hat{k}$ we have

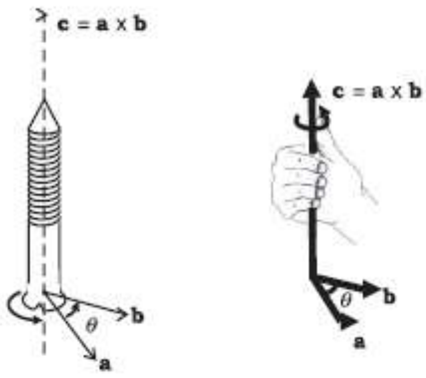
$$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$$

$$\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$$

- Given two vectors

Vector Product Of Two Vectors:

- A vector product of two vectors **a** and **b** is a vector **c** such that
- (i) magnitude of **c** = $c = ab \sin\theta$ where a and b are magnitudes of **a** and **b** and θ is the angle between the two vectors.
- (ii) **c** is perpendicular to the plane containing **a** and **b**.
- (iii) if we take a right handed screw with its head lying in the plane of **a** and **b** and the screw perpendicular to this plane, and if we turn the head in the direction from **a** to **b**, then the tip of the screw advances in the direction of **c**.
- **This right handed screw rule** is illustrated in Fig. a.



(a) (b)

- Alternately, if one curls up the fingers of right hand around a line perpendicular to the plane of the vectors **a** and **b** and if the fingers are curled up in the direction from **a** to **b**, then the stretched thumb points in the direction of **c**, as shown in Fig. b.

- A simpler version of the right hand rule is the following :

- Open up your right hand palm and curl the fingers pointing from **a** to **b**. Your stretched thumb points in the direction of **c**.

- It should be remembered that there are two angles between any two vectors **a** and **b** . In Fig (a) or (b)

- they correspond to θ (as shown) and (360^0– θ).
- While applying either of the above rules, the rotation should be taken through the smaller angle (<180^0) between **a** and **b**.
.
- Because of the cross used to denote the vector product, it is also referred to as cross product.

Properties Of Vector Product:

- The vector product is **not commutative**, i.e. **a** × **b** ≠ **b** × **a**
- The magnitude of both **a** × **b** and **b** × **a** is the **same** (*ab* sinθ);
- also, both of them are perpendicular to the plane of **a** and **b**.
- But the rotation of the right-handed screw in case of **a** × **b** is from **a** to **b**, whereas in
- case of **b** × **a** it is from **b** to **a**.
- This means the two vectors are in **opposite** directions.
- We have **a** × **b = −b** × **a**
- Both scalar and vector products are **distributive** with respect to vector addition.
-
- Thus, **a**.(**b** + **c**) = **a**.**b** + **a**.**c** **a** × (**b** + **c**) = **a** × **b** + **a** × **c**
- We may write **c** = **a** × **b** in the component form.
- For this we first need to obtain some elementary cross products:
- **a** × **a** = **0** (**0** is a null vector, i.e. a vector with zero magnitude)
- This follows since magnitude of **a** × **a** is a^2 sin0° = 0 .
- From this follow the results **ˆi × ˆi** = **0**, **ˆj × ˆj** = **0**, **kˆ × kˆ = 0**

- **ˆi** × **ˆj** = **kˆ** Note that the magnitude of **ˆi × ˆj** is sin90^0 or 1,
- Since **ˆi** and **ˆj** both have unit magnitude and the angle between them is 90^0.
- Thus, **ˆi × ˆj** is a unit vector.
- A unit vector perpendicular to the plane of **ˆi** and **ˆj** and related to them by the right hand screw rule is **ˆk**
- . Similarly **ˆj** × **kˆ** = **ˆi** and **kˆ** × **ˆi** = **ˆj**
- From the rule for commutation of the cross product, it follows:
- **ˆj × ˆi = −kˆ**, **kˆ × ˆj = −ˆi**, **ˆi** × **kˆ = −ˆj**
- if **ˆi**, **j**,**kˆ** occur cyclically in the above vector product relation, the vector product is positive.

if **ˆi**, **ˆj**,**kˆ** do not occur in cyclic order, the vector product is negative.

$$\mathbf{a}\times\mathbf{b} = (a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}} + a_z\hat{\mathbf{k}}) \times (b_x\hat{\mathbf{i}} + b_y\hat{\mathbf{j}} + b_z\hat{\mathbf{k}})$$
$$= a_x b_y\hat{\mathbf{k}} - a_x b_z\hat{\mathbf{j}} - a_y b_x\hat{\mathbf{k}} + a_y b_z\hat{\mathbf{i}} + a_z b_x\hat{\mathbf{j}} - a_z b_y\hat{\mathbf{i}}$$
$$= (a_y b_z - a_z b_y)\hat{\mathbf{i}} + (a_z b_x - a_x b_z)\hat{\mathbf{j}} + (a_x b_y - a_y b_x)\hat{\mathbf{k}}$$

- The expression for **a** × **b** can be put in a determinant form which is easy to remember.

$$\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

For example: ,

- moment of a force is a vector product of lever arm and force . $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$
- angular momentum is a vector product of position vector and momentum. $\mathbf{l} = \mathbf{r} \times \mathbf{p}$

THANK YOU

BIBLIOGRAPHY

•NCERT TEXT BOOK- CLASS XII-PHYSICS

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