Class XI - MATHEMATICS

Chapter 3 – TRIGONOMETRIC FUNCTIONS

Module – 3/3

By Smt. Mini Maria Tomy
PGT Mathematics
AECS KAIGA

Distance Learning Programme: An initiative by AEES, Mumbai

Learning Outcome

In this module we are going to learn about

- **Trigonometric Functions of Sum and Difference of Two Angles.**
- **Representation of T-ratios of multiples of an angle in terms of T-ratios of an angle.**
- **Representation of product of T-ratios as sum or difference of T-ratios.**
- **Representation of sum or difference of T-ratios as product of T-ratios.**

T- Functions of Sum and Difference of Two Angles.

1). $\cos(x + y) = \cos x \cos y - \sin x \sin y$

In figure, $\Delta P_1OP_3 \cong \Delta P_2OP_4$.

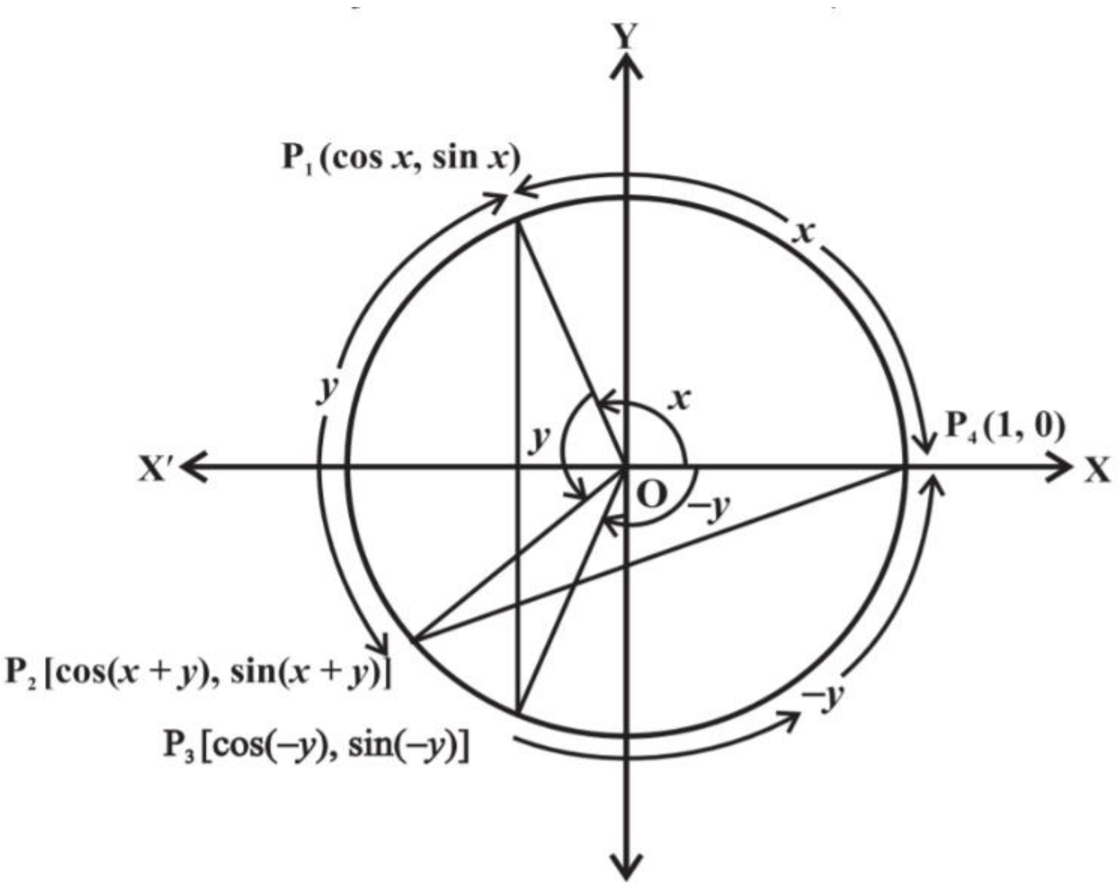
Therefore, $P_1 P_3 = P_2 P_4$

$P_1P_3{}^2 = [\cos x - \cos(-y)]^2 + [\sin x - \sin(-y)]^2$

$= 2 - 2(\cos x \cos y - \sin x \sin y)$ ......(1)

$P_2P_4{}^2 = [1 - \cos(x + y)]^2 + [0 - \sin(x + y)]^2$

$= 2 - 2\cos(x + y)$...................(2)



From (1) and (2) we get,

$\cos(x + y) = \cos x \cos y - \sin x \sin y$

T- Functions of Sum and Difference of Two Angles.

1) $\cos(x + y) = \cos x \cos y - \sin x \sin y$

2) $\cos(x - y) = \cos x \cos y + \sin x \sin y$

3) $\sin(x + y) = \sin x \cos y + \cos x \sin y$

4) $\sin(x - y) = \sin x \cos y - \cos x \sin y$

i	$\cos\left(\frac{\pi}{2} - x\right) = \sin x$	ix	$\cos\left(\frac{3\pi}{2} - x\right) = -\sin x$
ii	$\sin\left(\frac{\pi}{2} - x\right) = \cos x$	x	$\sin\left(\frac{3\pi}{2} - x\right) = -\cos x$
iii	$\cos\left(\frac{\pi}{2} + x\right) = -\sin x$	xi	$\cos\left(\frac{3\pi}{2} + x\right) = \sin x$
iv	$\sin\left(\frac{\pi}{2} + x\right) = \cos x$	xii	$\sin\left(\frac{3\pi}{2} + x\right) = -\cos x$
v	$\cos(\pi - x) = -\cos x$	xiii	$\cos(2\pi - x) = \cos x$
vi	$\sin(\pi - x) = \sin x$	xiv	$\sin(2\pi - x) = -\sin x$
vii	$\cos(\pi + x) = -\cos x$	xv	$\cos(2\pi + x) = \cos x$
viii	$\sin(\pi + x) = -\sin x$	xvi	$\sin(2\pi + x) = \sin x$

If none of the angles x, y and (x + y) is an odd multiple of $\frac{\pi}{2}$, then

i) $\tan(x+y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$

ii) $\tan(x-y) = \dfrac{\tan x - \tan y}{1 + \tan x \tan y}$

Proof : i) $\tan(x+y) = \dfrac{\sin(x+y)}{\cos(x+y)}$

$$= \frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y} = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

Replacing y by -y we can prove that , $\tan(x-y) = \dfrac{\tan x - \tan y}{1 + \tan x \tan y}$

If none of the angles *x*, *y* and (*x* + *y*) is a multiple of p, then

i) $\cot(x+y) = \dfrac{\cot x \cot y - 1}{\cot y + \cot x}$ ii) $\cot(x-y) = \dfrac{\cot x \cot y + 1}{\cot y - \cot x}$

Proof: $\cot(x+y) = \dfrac{\cos(x+y)}{\sin(x+y)}$

$$= \frac{\cos x \cos y - \sin x \sin y}{\sin x \cos y + \cos x \sin y} = \frac{\cot x \cot y - 1}{\cot y + \cot x}$$

Replacing y by -y we can prove that , $\cot(x-y) = \dfrac{\cot x \cot y + 1}{\cot y - \cot x}$

Representation of T-ratios of multiples of an angle in terms of T-ratios of an angle.

i). $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1$

$= 1 - 2\sin^2 x$

$= \frac{1 - \tan^2 x}{1 + \tan^2 x}$, $x \neq (2n+1)\frac{\pi}{2}$, $n \in Z$

ii) $\sin 2x = 2\sin x \cos x = \frac{2\tan x}{1 + \tan^2 x}$, $x \neq (2n+1)\frac{\pi}{2}$, $n \in Z$

iii) $\tan 2x = \dfrac{2\tan x}{1+\tan^2 x}$, $x \neq (2n+1)\dfrac{\pi}{2}$, $n \in Z$

iv) $\sin 3x = 3\sin x - 4\sin^3 x$

v) $\cos 3x = 4\cos^3 x - 3\cos x$

vi) $\tan 3x = \dfrac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$

Representation of product of T-ratios as sum or difference

i) $2 \cos x \cos y = \cos (x + y) + \cos (x - y)$

ii) $-2 \sin x \sin y = \cos (x + y) - \cos (x - y)$

iii) $2 \sin x \cos y = \sin (x + y) + \sin (x - y)$

iv) $2 \cos x \sin y = \sin (x + y) - \sin (x - y)$.

Representation of sum or difference of T-ratios as product

i) $\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

ii) $\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$

iii) $\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

iv) $\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$

Example 1:

Find the value of sin 75°.

Solution: We have, $\sin 75° = \sin(45° + 30°)$

$$= \sin 45° \cos 30° + \cos 45° \sin 30°$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$= \frac{\sqrt{3} + 1}{2\sqrt{2}}$$

Example 2

Show that, tan 3x tan 2x tan x = tan 3x – tan 2x – tan x

Proof: **tan 3x = tan (2x + x)**

$$= \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$$

or, tan 3x$(1 - \tan 2x \tan x) = \tan 2x + \tan x$

or, tan 3x – tan 3x tan 2x tan x = tan 2x + tan x

or, tan 3x – tan 2x – tan x = tan 3x tan 2x tan x

or, tan 3x tan 2x tan x = tan 3x – tan 2x – tan x.

Example 3:

Prove that $\frac{\cos 7x+\cos 5x}{\sin 7x-\sin 5x} = \cot x.$

Proof : $\frac{\cos 7x + \cos 5x}{\sin 7x - \sin 5x} = \frac{2\cos\left(\frac{7x+5x}{2}\right)\cos\left(\frac{7x-5x}{2}\right)}{2\cos\left(\frac{7x+5x}{2}\right)\sin\left(\frac{7x-5x}{2}\right)}$

$$= \frac{2\cos 6x.\cos x}{2\cos 6x.\sin x} = \cot x$$

What have we learned today?

- Trigonometric Functions of Sum and Difference of Two Angles.
- Representation of T-ratios of multiples of an angle in terms of T-ratios of an angle.
- Representation of product of T-ratios as sum or difference of T-ratios.
- Representation of sum or difference of T-ratios as product of T-ratios.

THANK YOU