ATOMIC ENERGY EDUCATION SOCIETY

Distant Learning Programme Class XI

Subject: Physics

Hand out study Material

Chapter: Unit and Measurement (Module 3/4)

Contents

- Propagation and combination of errors in addition, substraction,multiplication,division & power raised to quantity.
- Significant figures and their rules.
- Rounding off of uncertain digit in significant figure.
- Rules for Arithmetic Operations with Significant Figures**.**

Propagation and Combination of Errors

- **a) Combination of Error of a sum :** Suppose two physical quantities A and B have measured values $A \pm \Delta A$, $B \pm \Delta B$ respectively where ΔA and ΔB are their absolute errors. We wish to find the error ΔZ in the sum

$Z = A + B$.

We have by addition, $Z \pm \Delta Z = (A \pm \Delta A) + (B \pm \Delta B)$

The maximum possible error in Z is

$$\pm \Delta Z = \pm (\Delta A + \Delta B)$$

- **Hence the rule : When two quantities are added, the absolute error in the final result is the sum of the absolute errors in the individual quantities**

- **B) Combination of Error of difference**
- Suppose two physical quantities *A and B have* measured values $A \pm \Delta A$, $B \pm \Delta B$ *respectively* where ΔA *and* ΔB *are their* absolute *errors. We* wish to find the error ΔZ *in the difference*
- $Z = A - B$.

  We have by subtraction, $Z \pm \Delta Z = (A \pm \Delta A)-(B \pm \Delta B)$ $Z \pm \Delta Z = (A - B) \pm \Delta A \pm \Delta B$

  The maximum possible error in *Z is*

$$\pm \Delta Z = \pm (\Delta A + \Delta B)$$

- **Hence the rule: When two quantities are subtracted, the absolute error in the final result is the sum of the absolute errors in the individual quantities.**

- **(C) Combination of Error of a product**

- Suppose $Z = AB$ and the measured values of A and B are $A \pm \Delta A$ and $B \pm \Delta B$. Then
- $Z \pm \Delta Z = (A \pm \Delta A)(B \pm \Delta B)$

  $= AB \pm B\Delta A \pm A\Delta B \pm \Delta A\Delta B$.

  Dividing LHS by Z and RHS by AB we have, $1 \pm (\Delta Z/Z) = 1 \pm (\Delta A/A) \pm (\Delta B/B) \pm (\Delta A/A)(\Delta B/B)$.

  Since ΔA and ΔB are small, we shall ignore their product. Hence the maximum relative error

$$\pm \Delta Z/Z = (\Delta A/A) + (\Delta B/B).$$

- **Hence the rule: When two quantities are multiplied or divided, the relative error in the result is the sum of the relative errors in the multipliers.**

- **(D) Combination of Error of a quotient**

$$\therefore\ x \pm \Delta x = \frac{a \pm \Delta a}{b \pm \Delta b}$$

$$\therefore\ x \pm \Delta x = (a \pm \Delta a)(b \pm \Delta b)^{-1}$$

$$\therefore\ x \pm \Delta x = a\left(1 \pm \frac{\Delta a}{a}\right)(b)^{-1}\left(1 \pm \frac{\Delta b}{b}\right)^{-1}$$

$$\therefore\ x \pm \Delta x = \frac{a}{b}\left(1 \pm \frac{\Delta a}{a}\right)\left(1 \pm \frac{\Delta b}{b}\right)^{-1}$$

Expanding binomially

$$x \pm \Delta x = \frac{a}{b}\left(1 \pm \frac{\Delta a}{a}\right)\left(1 \mp \frac{\Delta b}{b} \pm \text{terms containing higher powers of } \frac{\Delta b}{b}\right)$$

$$\therefore\ \pm\frac{\Delta x}{x} = \pm\frac{\Delta a}{a} \pm \frac{\Delta b}{b}$$

$$\therefore\ \frac{\Delta x}{x} = \frac{\Delta a}{a} + \frac{\Delta b}{b}$$

- **(E) Error in case of a measured quantity raised to a power**

  Suppose $Z = A^2$

  Then, $\Delta Z/Z = (\Delta A/A) + (\Delta A/A) = 2\,(\Delta A/A)$.

- Hence, the relative error in A^2 is two times the error in A.

  In general, if $Z = A^p\ B^q / C^r$ Then,

$$\pm\ \Delta Z/Z = p\,(\Delta A/A) + q\,(\Delta B/B) + r\,(\Delta C/C).$$

- **Hence the rule: The relative error in a physical quantity raised to the power k is the k times the relative error in the individual quantity.**

38. A physical quantity X is related to four measurable quantities a, b, c and d as follows $X = a^2 b^3 c^{5/2} d^{-2}$. The percentage error in the measurement of a, b, c and d are 1%, 2%, 3% and 4%, respectively. What is the percentage error in quantity X? If the value of X calculated on the basis of the above relation is 2.763, to what value should you round off the result.

Sol. Percentage error in quantity X is given by, $\frac{\Delta x}{x} \times 100$

According to the problem, physical quantity is $X = a^2 b^3 c^{5/2} d^{-2}$

percentage error in $a = \left(\frac{\Delta a}{a} \times 100\right) = 1\%$

percentage error in $b = \left(\frac{\Delta b}{b} \times 100\right) = 2\%$

percentage error in $c = \left(\frac{\Delta c}{c} \times 100\right) = 3\%$

percentage error in $d = \left(\frac{\Delta d}{d} \times 100\right) = 4\%$

Maximum percentage error in X is

$$\frac{\Delta X}{X} \times 100 = \pm\left[2\left(\frac{\Delta a}{a} \times 100\right) + 3\left(\frac{\Delta b}{b} \times 100\right) + \frac{5}{2}\left(\frac{\Delta c}{c} \times 100\right) + 2\left(\frac{\Delta d}{d} \times 100\right)\right]$$

$$= \pm\left[2(1) + 3(2) + \frac{5}{2}(3) + 2(4)\right]\%$$

$$= \pm\left[2 + 6 + \frac{15}{2} + 8\right] = \pm 23.5\%$$

$\therefore$ Percentage error in quantity $X = \pm 23.5\%$

Mean absolute error in $X = \pm 0.235 = \pm 0.24$ (rounding-off upto two significant digits)

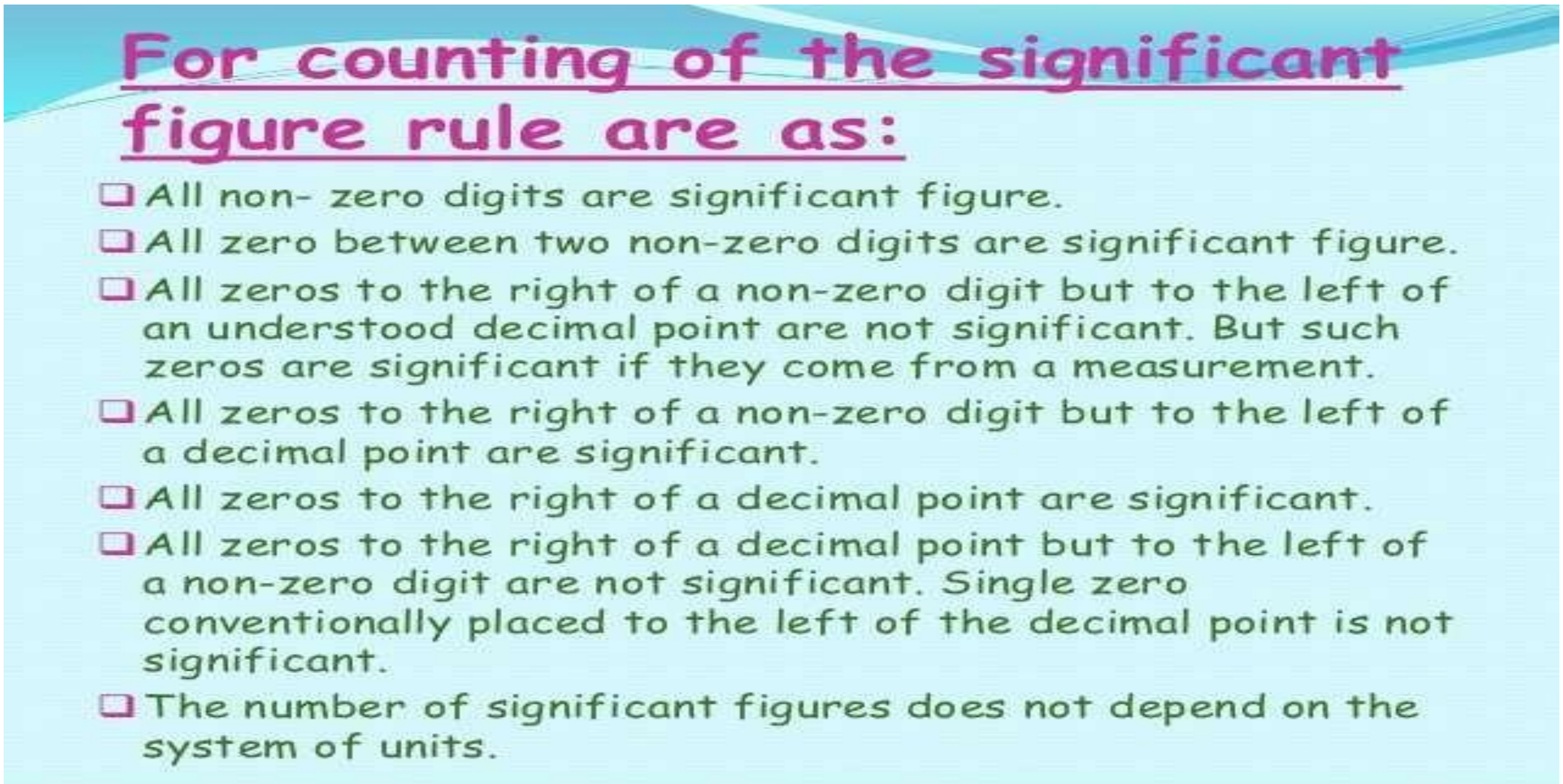
On the basis of these values, the value of X should have two significant digits only.

$\therefore$ $X = 2.8$

Significant figures

- In any measured quantity the reliable digits plus the first uncertain digit are known as **significant digits or significant figures.**
- **For Ex.** If we say the period of oscillation of a simple pendulum is 1.62 s, the digits 1 and 6 are reliable and certain, while the digit 2 is uncertain. More no. of significant figure means more precise that measured quantity.

Rules for determining significant figure



Rules for rounding off uncertain digit

- Insignificant digit to be dropped is more than 5 Preceding digit is raised by 1.
  Ex. Number : 3.137 Result : 3.14
- In significant digit to be dropped is less than 5 Preceding digit is left unchanged
- Ex. Number : 3.132 Result : 3.13 I
- nsignificant digit to be dropped is equal to 5 If preceding digit is even, it is left unchanged.
- Ex. Number : 3.12 Result : 3.12
- Insignificant digit to be dropped is equal to 5 If preceding digit is odd, it is raised by 1.
- Ex. Number : 3.135 Result : 3.14

Rules for ArithmeticOperations with Significant Figures

- **In multiplication or division, the final result should retain as many significant figures as are there in the original number with the least significant figures.**
- In the example given, density should be reported to *three significant figures.*
  Density = (4.237g/2.51 cm) = 1.69 g Cm^3
- **In addition or subtraction, the final result should retain as many decimal places as are there in the number with the least decimal places.**
- For example, the sum of the numbers 436.32 g, 227.2 g and 0.301 g by mere arithmetic addition, is 663.821 g. But the least precise measurement (227.2 g) is correct to only one decimal place. The final result should, therefore, be rounded off to 663.8 g.

REFERENCES:

NCERT XI CLASS WIKIPEDIA

CONCEPT OF PHYSICS BY H C VERMA

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