Class XI- MATHEMATICS
Chapter-2 : RELATIONS and FUNCTIONS
Hand out of Module 2/2

Learning Outcome:

In this module we are going to learn about

- Functions
- Domain, Co-domain and Range of a Function
- Types of functions

Function: A relation f from a set A to a set B is said to be a function if every element of set A has one and only one image in set B.

If f is a function from A to B and $(a, b) \in f$, then $f(a) = b$, where 'b' is called the image of 'a' under f and 'a' is called the pre image of 'b' under f.

The function f from A to B is denoted by $f : A \to B$

Example 1:

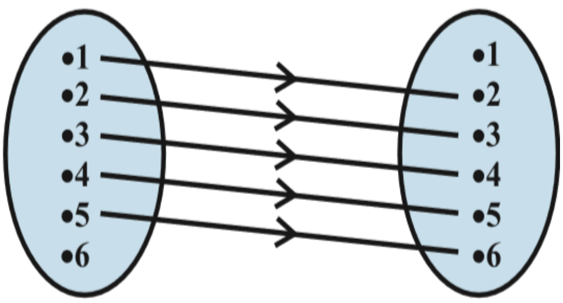
Let A = {1, 2, 3, 4, 5, 6}. Define a relation R from A to A by

$R = \{(x , y) : y = x + 1\}$ (i) Depict this relation using an arrow diagram. (ii) Write down the domain, codomain and range of R.

(iii) Is the given relation a function? Give reason.

Solution:

(i). Following figure depicts the relation $R = \{(x , y) : y = x + 1\}$.



(ii) Domain = {1, 2, 3, 4, 5, 6}. Co-domain = {1, 2, 3, 4, 5, 6}, Range = {2, 3, 4, 5, 6}

(iii) Since the element '6'in the domain is not having an image, this relation is not a function.

Example 2:

Let **N** be the set of natural numbers and the relation R be defined on N such that

R = {$(x, y) : y = 2x, x, y \in$ **N**}.

What is the domain, codomain and range of R? Is this relation a function?

Solution: The domain of R is the set of natural numbers **N**. The codomain is also **N**.

The range is the set of even natural numbers.

Since every natural number *n* has one and only one image, this relation is a function.

Example 3 :

Examine each of the following relations given below and state in each case, giving reasons whether it is a function or not?

(i). R = {(2, 1), (3, 1), (4, 2)}

(ii). R={(2, 2), (2, 4), (3, 3), (4, 4)}

(iii). R = {(1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 7)}

Solution (i) Since 2, 3, 4 are the elements of domain of R having their unique images,

this relation R is a function.

(ii) Since the same first element 2 corresponds to two different images 2 and 4, this relation is not a function.

(iii) Since every element has one and only one image, this relation is a function.

Note:

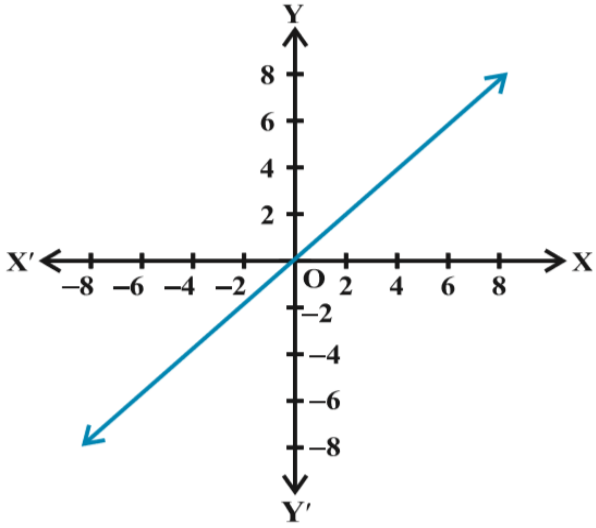
1) A function which has either R or one of its subsets as its range is calleda real valued function.

2) A function which has either R or one of its subsets as domain & range is calleda real function.

Some functions and their graphs

(i). Identity function:

Let **R** be the set of real numbers. Define the real valued function f : R→ R by y = f(x) = x for each x ϵ **R**. Such a function is called the identity function. Here the domain and range of f are **R**. The graph is a straight line. It passes through the origin.

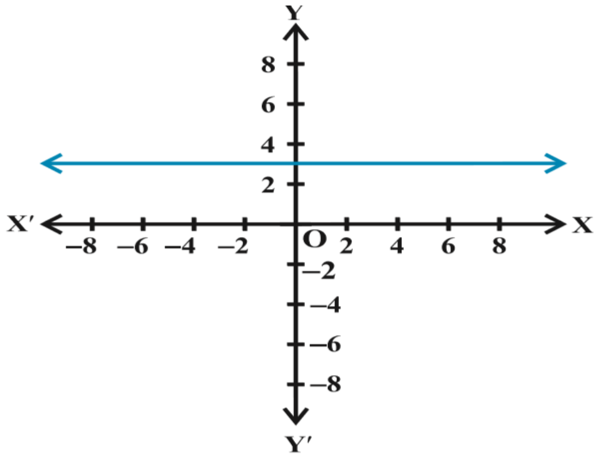


$f(x) = x$

(ii) Constant function:

Define the function f : R → R by $y = f(x) = c$, x ϵ**R** where c is a constant and each x ϵ **R**. Here domain of f is **R** and its range is {c}.

The graph is a line parallel to x-axis. For example, if $f(x)$ =3 for each x ϵ **R**, then its graph will be a line parallel to x-axis as shown in the adjacent figure.



$f(x) = 3$

(iii) Polynomial function:

A function f: R → R defined by f (x) = $a_0 + a_1 x + a_2 x^2 + ...+ a_n x^n$, where n is a non-negative integer and $a_0, a_1, ... a_n$ ϵ **R** is called a polynomial function.

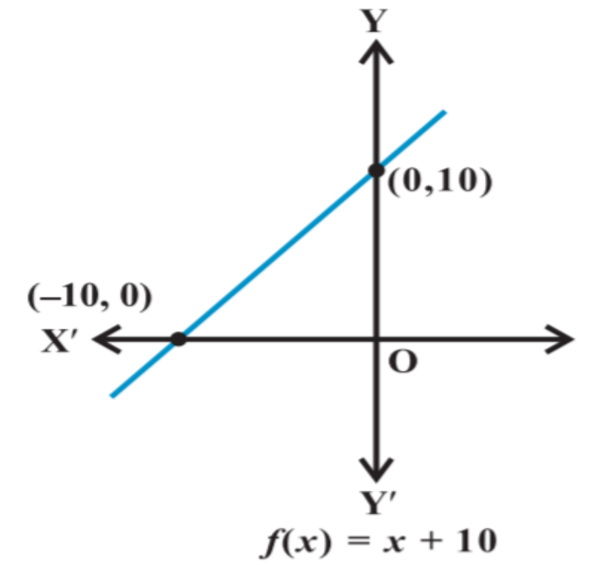
Note:

1). The function f defined by f(x) = ax + b, x ∈ R, is called linear function, where a , b ϵ **R**

2) The function f defined by f(x) = ax^2 + bx + c, x ∈ R, is called quadratic function, where a , band c ϵ **R**

Example 1:

Let R be the set of real numbers. Define the real function f: R→R by f(x) = x + 10 and sketch the graph of this function.



Solution :

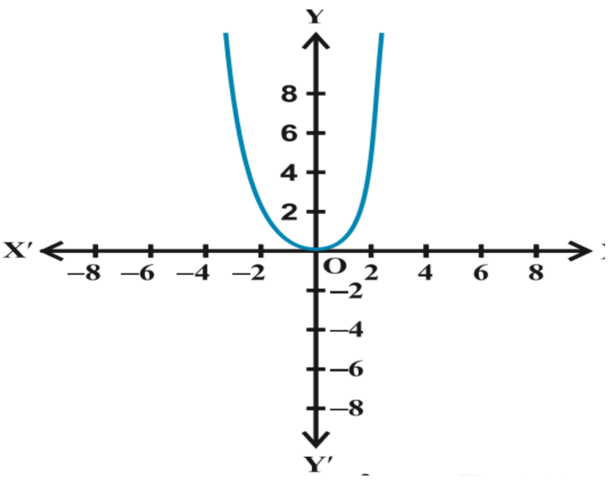
Domain of f = {x : x ∈ R}.

Range of f = {x^2 : x ∈ R}.

x	-10	-1	0	1	2
y = f(x)	**0**	**9**	**10**	**11**	**12**

Example 2:

Draw the graph of the function f : R → R defined by $f(x) = x^2$ **, x ∈ R.**



Solution :

Domain of f = {x : x ∈ R}.

Range of f = {x^2 : x ∈ R}.

x	-2	-1	0	1	2
y =f(x)	**4**	**1**	**0**	**1**	**4**

The graph of **f (x) = x^2** is given in the adjacent figure

Example 3:

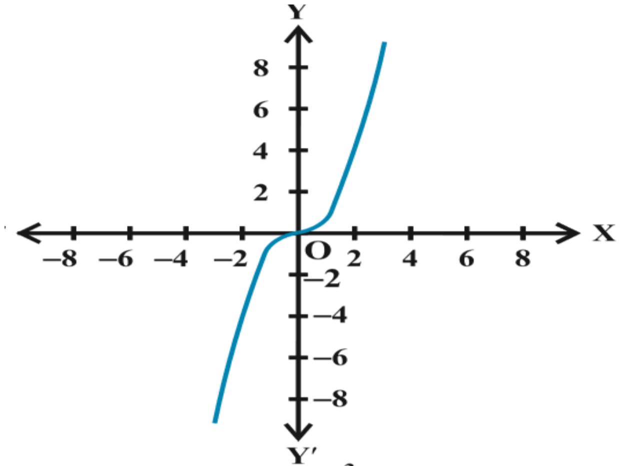
Draw the graph of the function f : R → R defined by f (x) = x^3, x ∈ R.

Solution : Domain of f = {x : x ∈ R}.

Range of f = {x^3 : x ∈ R}.

X	-2	-1	0	1	2
y = f(x)	-8	-1	0	1	8

The graph of **f (x) = x^3** is given in the adjacent figure



$f(x) = x^3$

(iv). <u>Rational functions</u>:

A function f: R → R is said to be rational function if

f(x) = $\frac{g(x)}{h(x)}$, where g(x) and h(x) are polynomial functions of x defined in a domain, where h(x) ≠ 0.

Example:

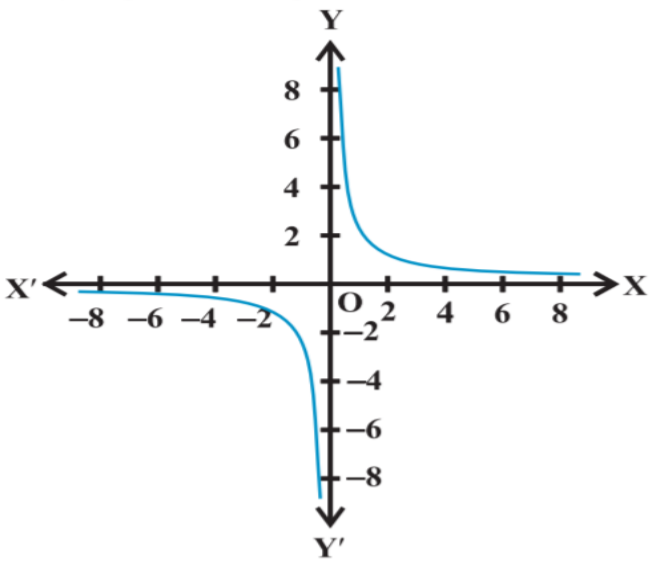
Consider the real valued function f : R – {0} → R defined by f (x) = $\frac{1}{x}$, x ∈ R –{0}. What is the domain and range of this function?

Draw the graph of this function.

Solution:

The domain is all real numbers except 0 and its range is also all real numbers except 0.

The graph of *f* is given in the adjacent figure.



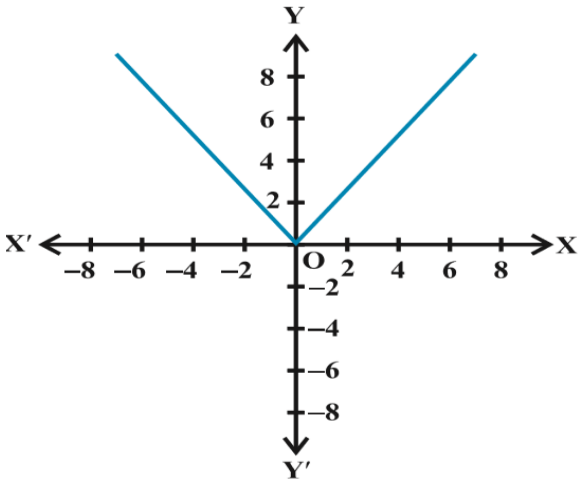
$f(x) = \frac{1}{x}$

(v). <u>The Modulus function</u>:

The function f: R → R defined by ***f*(*x*) = |*x*|** for each $x \in$ **R** is called *modulus function*. For each non-negative value of *x*, *f*(*x*) is equal to *x*. But for negative values of *x*, the value of *f*(*x*) is the negative of the value of *x*.

i.e. f(x) = $\begin{cases} x, & if\ x \geq 0 \\ -x, & if\ x < 0 \end{cases}$

The graph of the modulus function is given in the adjacent figure.

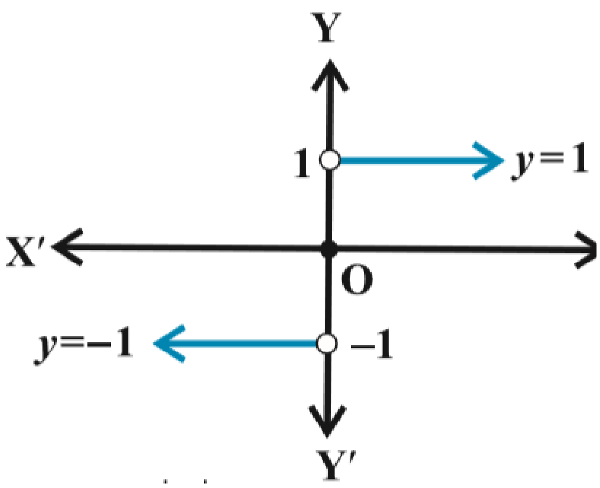


f(*x*) = |*x*|

(vi).<u>Signum function</u>:

The function f: R → R defined by

f(*x*) = $\begin{cases} 1, & if\ x > 0 \\ 0, & if\ x = 0 \\ -1, & if\ x < 0 \end{cases}$ is called the signum function. The domain of the signum function is R and the range is the set {–1, 0, 1}. The graph of the signum function is shown in the adjacent figure.



(vii). <u>Greatest integer function:</u>

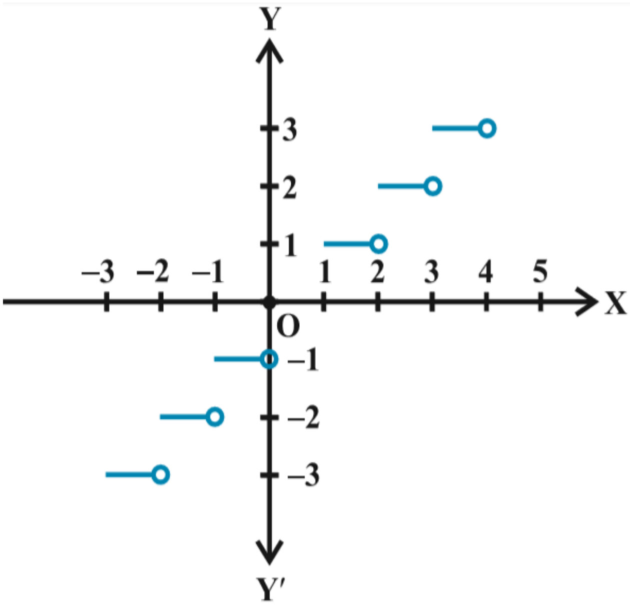
The function f: R → R defined by $f(x) = [x]$, $x \in \mathbf{R}$ assumes the
value of the greatest integer, less than or equal to x.
Such a function is called the greatest integer function.
From the definition of $[x]$, we can see that

$[x] = -1$ for $-1 \leq x < 0$

$[x] = 0$ for $0 \leq x < 1$

$[x] = 1$ for $1 \leq x < 2$

$[x] = 2$ for $2 \leq x < 3$ and so on. The graph of the function is shown in the adjacent figure.



For greatest integer function,
Domain = R and Range = Z

Example:

Find the domain of the function f(x) = $\frac{x^2+3x+5}{x^2-5x+6}$

Solution : Since $x^2 - 5x + 6$ = (x – 2) (x –3), the function f is defined for all real numbers except at x = 2 and x = 3. Hence the domain of f is R – {2, 3}.

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