Class XI - MATHEMATICS

Chapter 2 – RELATIONS AND FUNCTIONS

Module – 1/2

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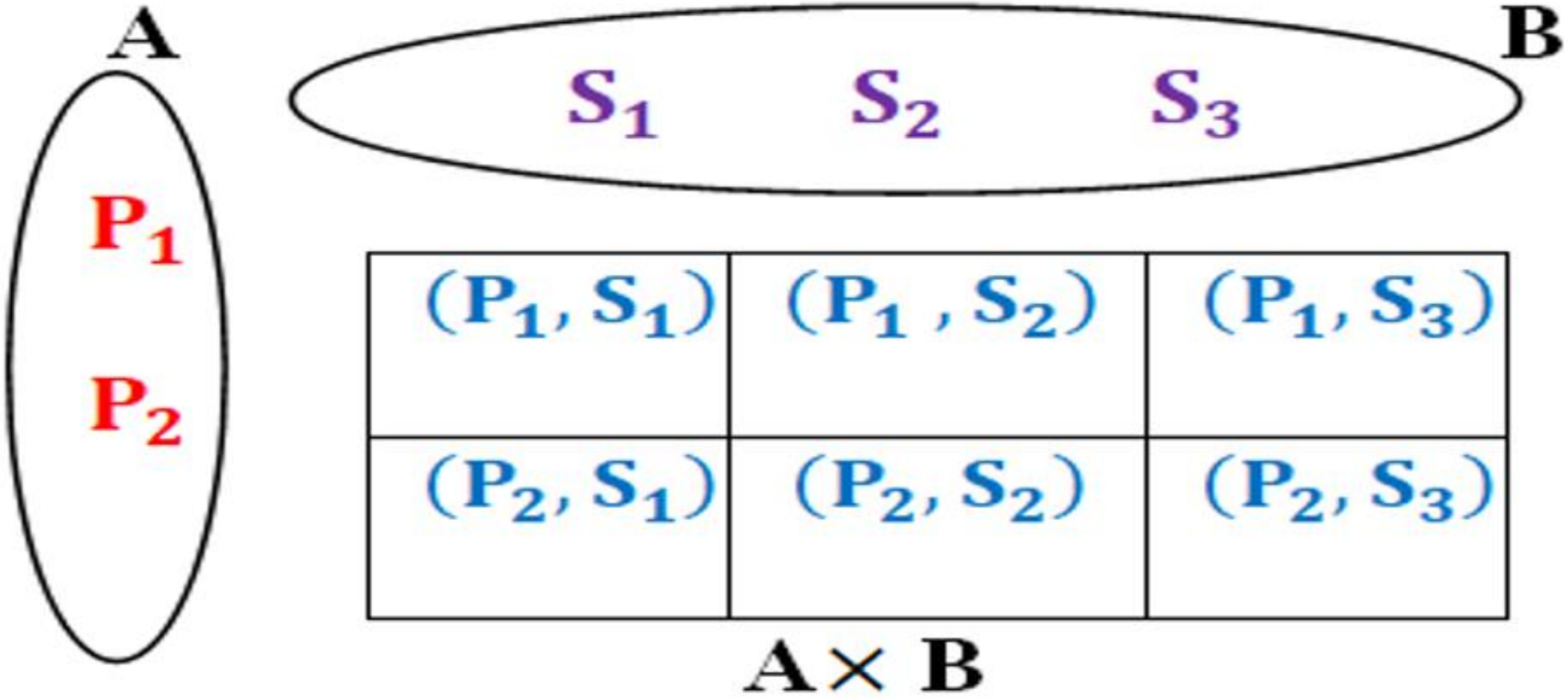
Distance Learning Programme: An initiative by AEES, Mumbai

Cartesian Product of Sets



Cartesian Products of Sets

Given two non-empty sets A and B.

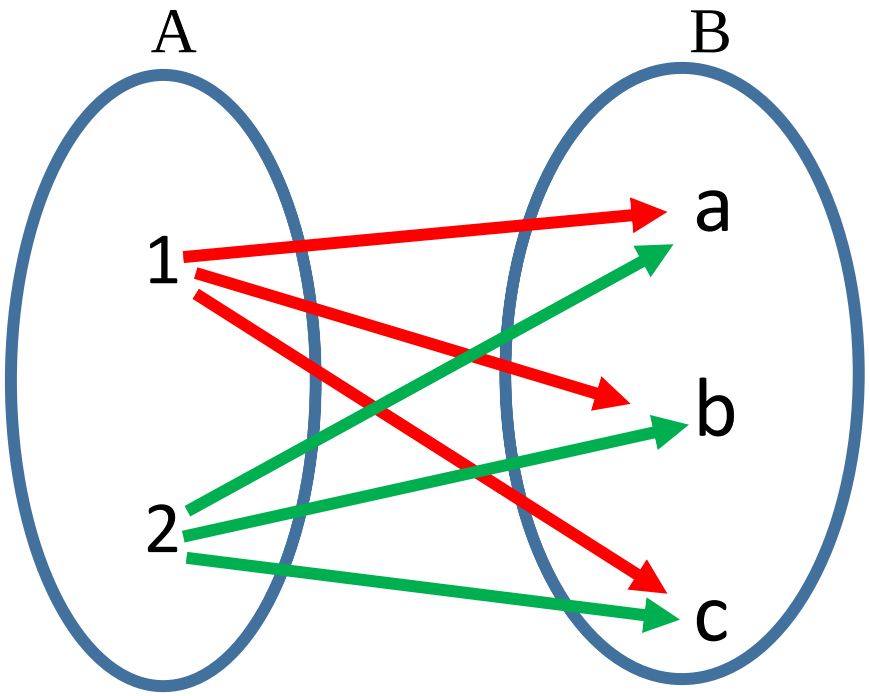
The cartesian product $A \times B$ is the set of all ordered pairs of elements from A and B,

i.e., $A \times B = \{ (a, b) : a \in A, \; b \in B \}$

Example:

Let A ={1,2} and B ={ a, b, c}.

Find A × B.



Solution:

A× B = {(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)}

$\emptyset$

Note :

- **If either A or B is an empty set, then , $A \times B = \emptyset$**
- **If A and B are non-empty sets and either A or B is an infinite set, then so is $A \times B$.**
- **Two ordered pairs are equal, if and only if the corresponding first elements are equal and the second elements are also equal.**

Example: If , $(a - 3, b + 2) = (4, -2)$, find the values of a and b.

$a - 3 = 4$ and $b + 2 = -2$. Therefore, $a = 7$ and $b = -4$.

Note

- if $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$.
- In general, $A \times B \neq B \times A$
- $A \times A \times A = \{(a, b, c) : a, b, c \in A\}$. Here (a, b, c) is called an ordered triplet.
- $A \times (B \cap C) = (A \times B) \cap (A \times C)$ and

  $A \times (B \cup C) = (A \times B) \cup (A \times C)$

- **The Cartesian product $R \times R = \{(x, y) : x, y \in R\}$ represents the coordinates of all the points in two dimensional space.**
- **The cartesian product $R \times R \times R = \{(x, y, z) : x, y, z \in R\}$ represents the coordinates of all the points in three-dimensional space.**

Example :**If $P = \{a, b\}$ and $Q = \{x, y\}$, find $P \times Q$ and $Q \times P$.**

Are these two products equal ?

Solution: $P \times Q = \{(a, x), (a, y), (b, x), (b, y)\}$

and $Q \times P = \{(x , a), (x, b), (y, a), (y, b)\}$

The pair (a, x) is not equal to the pair (x, a).

Therefore $P \times Q \neq Q \times P$.

Example 4: Let $A = \{1, 2\}$, $B = \{3, 4\}$, $C = \{4, 5\}$

Verify that $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

Solution: $B \cap C = \{4\}$.

Therefore, $A \times (B \cap C) = \{1,2\}\times\{4\} = \{(1,4), (2,4)\}$………(1)

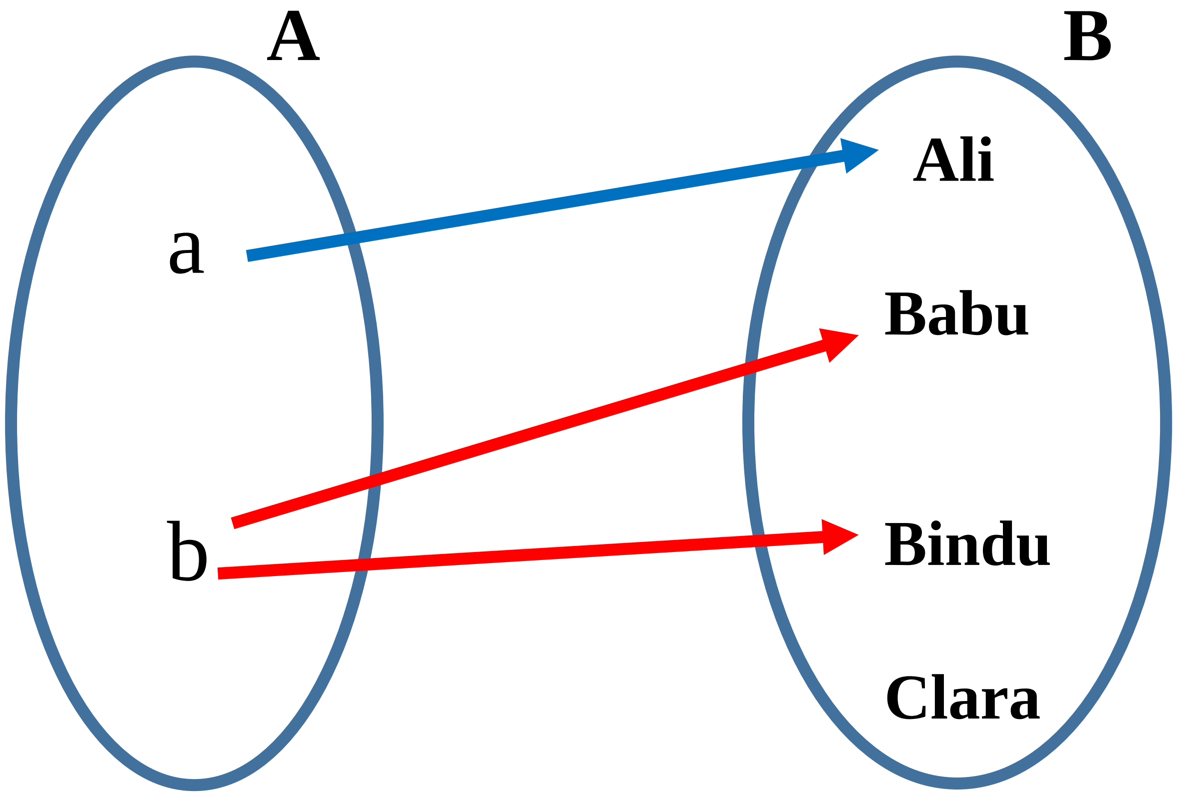
Also, $A\times B = \{(1, 3),(1,4), (2,3),(2,4)\}$,

$A\times C = \{(1, 4),(1,5), (2,4), (2,5)\}$……………………(2)

Therefore, $(A \times B) \cap (A \times C) = \{(1,4), (2,4)\}$

Hence, from (1) & (2), we get, $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

RELATIONS



R = {(a, Ali), (b, Babu), (b, Bindu)}

Relation:

A relation R from a non-empty set A to a non-empty set B is a subset of the cartesian Product A × B.

The subset is derived by describing a relationship between the first element and the second element of the ordered pairs in A × B. The second element is called the image of the first element.

DOMAIN, CO-DOMAIN & RANGE OF A RELATION

Domain : The set of all first elements of the ordered pairs in a relation R from a set A to a set B is called the domain of the relation R.

Codomain : The whole set B is called the codomain of the relation R.

Range : The set of all second elements in a relation R from a set A to a set B is called the range of the relation R.

NOTE

(i). range $\subset$ codomain.

(ii). A relation may be represented algebraically either by Roster method or by Set- builder method.

(iii). An arrow diagram is a visual representation of a relation.

(iv). If $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$. and the total number of relations from A to B is 2^{pq}.

(v). A relation R from A to A is also stated as a relation on A.

EXAMPLE:

Let A = {1, 2, 3, 4, 5}. Define a relation R from A to A by

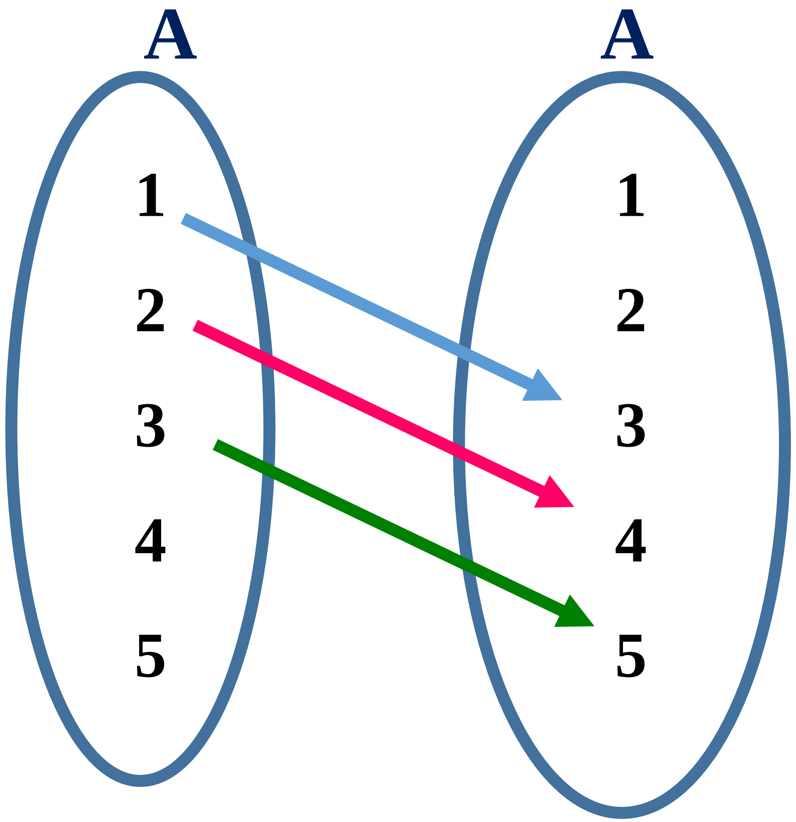
R = {(x , y) : y = x + 2 }

Then, R = {(1, 3), (2, 4), (3, 5)}.

domain of R ={1, 2, 3}

Co-domain of R = {1, 2, 3, 4, 5}

range of R = {3, 4, 5}



Example 2:

Let A = {1, 2} and B = {a, b, c}. Find the number of relations from A to B.

Solution:

We have, A × B = {(1, a), (1, b), (1,c), (2, a), (2, b),(2, c)}.

Since n (A×B) = 6. Therefore, the number of relations from A to B will be $2^6 = 64$.

What have we learned today?

- **Ordered pair:** **A pair of elements grouped together in a particular order.**
- **Cartesian product: Cartesian product of two sets A and B is given by $A \times B = \{(a, b): a \in A, b \in B\}$**
- $R \times R = \{(x, y): x, y \in R\}$ **and** $R \times R \times R = \{(x, y, z): x, y, z \in R\}$
- **If $(a, b) = (x, y)$, then $a = x$ and $b = y$.**
- **If $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$.**

- $A \times \varphi = \varphi$
- In general, $A \times B \neq B \times A$.
- **Relation:** A relation R from a set A to a set B is a subset of the cartesian product $A \times B$.
- **Domain:** The domain of R is the set of all first elements of the ordered pairs in a relation R.

Range: The range of the relation R is the set of all second elements of the ordered pairs in a relation R.

THANK YOU