Class XI - MATHEMATICS

Chapter 1- SETS

Hand out of Module-2/2

In this module we are going to learn about

- **Intervals as subset of R**
- **Power set**
- **Universal set**
- **Venn diagrams**
- **Union of sets**
- **Intersection of sets**

Intervals as subsets of R:

Open interval:

Let a, b ∈ **R** and a < b. Then the set of real numbers **{ x : a < x < b}** is called an open interval and is denoted by (a, b).

All the points between a and b belong to the open interval (a, b) but a, b themselves do not belong to this interval.

Closed interval: Let a, b ∈ **R** and a < b. Then the set of real numbers **{x : a ≤ x ≤ b**} is called closed interval and is denoted by [a, b].

We can also have intervals closed at one end and open at the other, i.e.,

[a, b) = {x : a ≤ x < b} is an open interval from a to b, including a but excluding b.

(a, b] = { x : a < x ≤ b } is an open interval from a to b including b but excluding a.

The set [0, ∞) defines the set of non-negative real numbers, while set (– ∞, 0) defines the set of negative real numbers.

The set (– ∞, ∞) describes the set of real numbers in relation to a line extending from – ∞ to ∞.

On real number line, various types of intervals described above as subsets of R, are shown in the following figure

Note: An interval contains infinitely many points.

For example, the set $\{x : x \in R, -5 < x \leq 7\}$, written in set-builder form, can be written in the form of interval as $(-5, 7]$ and the interval $[-3, 5)$ can be written in set builder form as $\{x : -3 \leq x < 5\}$.

Note: The number **(b – a)** is called the length of any of the intervals (a, b), [a, b], [a, b) or (a, b].

Power set :

The collection of all subsets of a set A is called the power set of A.

It is denoted by P(A). In P(A), every element is a set.

Example:

Let A = { 1, 2 }, then P(A) = { φ,{ 1 }, { 2 }, { 1,2 }}

Also, note that n [P (A)] = 4 = 2^2

Note: In general, if A is a set with n(A) = m, then it can be shown that **n [P(A)] = 2^m.**

Universal Set :

Usually, in a particular context, we have to deal with the elements and subsets of a basic set which is relevant to that particular context. This basic set is called the "Universal Set". The universal set is usually denoted by U, and all its subsets by the letters A, B, C, etc.

Example:

While studying the system of numbers, we are interested in the set of natural numbers and its subsets such as the set of all prime numbers, the set of all even numbers, and so forth. Here we can take the set of natural numbers as universal set.

Venn Diagrams

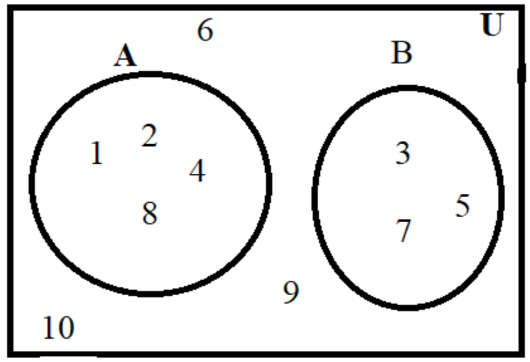
Most of the relationships between sets can be represented by means of diagrams which are known as Venn diagrams.

Venn diagrams are named after the English logician, **John Venn** (1834-1883).

In a Venn diagram, the universal set is represented usually by a rectangle and its subsets by circles. In Venn diagrams, the elements of the sets are written in their respective circles

Example:

Let U = {1,2,3, ..., 10} is the universal set of which A = {1,2,4,8} and B = {3,5,7} are subsets. These sets are represented using Venn Diagram in the adjacent figure.



<u>**Operations on Sets**</u>

1) <u>**Union of sets**</u>: The union of two sets A and B is the set C which consists of all those elements which are either in A or in B (including those which are in both).

   In symbols, we write. A ∪ B = { x : x ∈A or x ∈B }

Example 1:

Let A = { 2, 4, 6, 8} and B = { 6, 8, 10, 12}. Find A ∪ B.

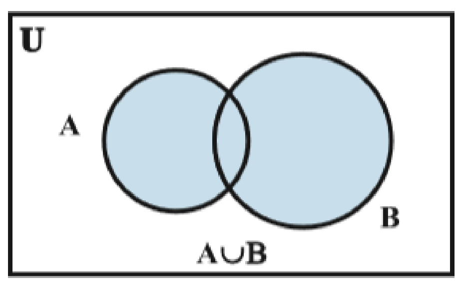
Solution: We have A ∪ B = { 2, 4, 6, 8, 10, 12}

Example 2: Let A = { a, e, i, o, u } and B = { a, i, u }. Show that A ∪ B = A

Solution: We have, A ∪ B = { a, e, i, o, u } = A.

Note : if B ⊂ A, then A ∪ B = A.

The union of two sets can be represented by a Venn diagram as shown in figure. The shaded portion in the figure represents A ∪ B.



<u>**Some Properties of the Operation of Union**</u>

i)A ∪ B = B ∪ A (Commutative law)

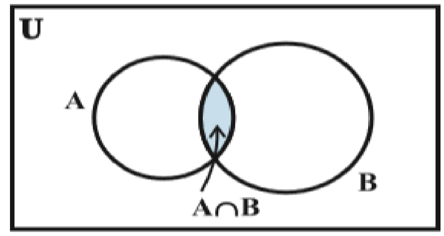
ii) (A ∪ B) ∪ C = A ∪ (B ∪ C) (Associative law)

iii) A ∪ φ = A (Law of identity element, φ is the identity of ∪)

iv) A ∪ A = A (Idempotent law)

v) U ∪ A = U (Law of U)

2) **Intersection of sets** : The intersection of two sets A and B is the set of all those elements which belong to both A and B. Symbolically, we write A ∩ B = {x : x ∈ A and x ∈ B}



Example :

Let A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and B = {2, 3, 5, 7 }. Find A ∩ B and hence show that A ∩ B = B.

Solution:

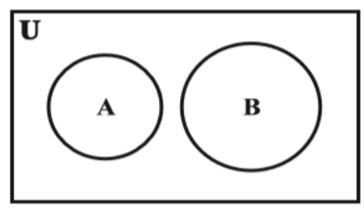
We have A ∩ B = { 2, 3, 5, 7 } = B.

Note 1:

If, B ⊂ A then, A ∩ B = B.

Note 2:

If A and B are two sets such that A ∩ B = φ, then A and B are called disjoint sets. The disjoint sets can be represented by means of Venn diagram as shown in the adjacent figure. Here, A and B are disjoint sets.



Some Properties of Operation of Intersection

i) A ∩ B = B ∩ A (Commutative law).

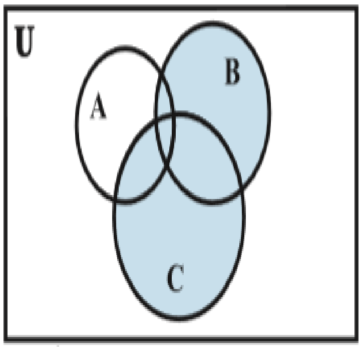
(ii) (A ∩ B) ∩ C = A ∩ (B ∩ C) (Associative law).

(iii) φ ∩ A = φ, U ∩ A = A (Law of φ and U).

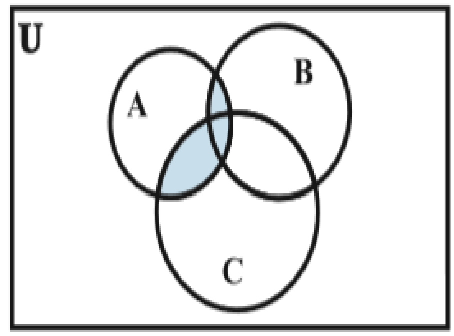
(iv) A ∩ A = A (Idempotent law)

(v) A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C) (Distributive law) i. e., ∩ distributes over ∪

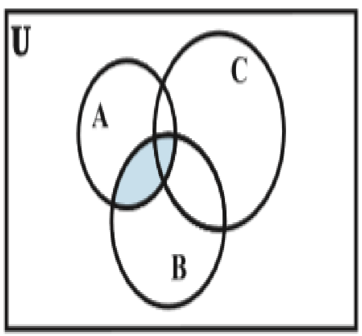
The following Venn diagrams shows the property, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$



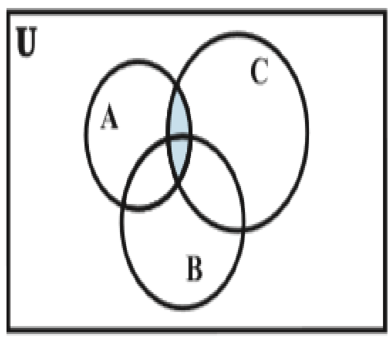
(i) $(B \cup C)$



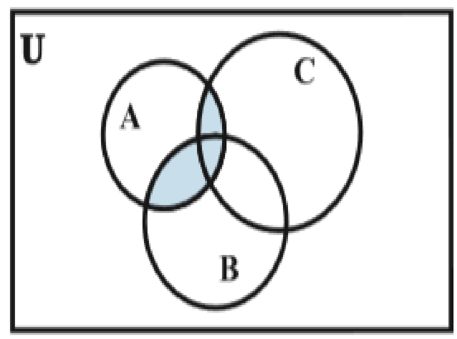
(ii) $A \cap (B \cup C)$



(iii) $(A \cap B)$



(iv) $(A \cap C)$



(v) $(A \cap B) \cup (A \cap C)$

<u>Practical Problems on Union and Intersection of Two Sets</u>

(i) Let A and B be finite sets. If $A \cap B = \varphi$, then $n (A \cup B) = n (A) + n (B)$.

(ii) if A and B are finite sets, then $n (A \cup B) = n (A) + n (B) – n (A \cap B)$

(iii) If A, B and C are finite sets, then

$n (A \cup B \cup C) = n (A) + n (B) + n (C) – n (A \cap B) – n (B \cap C) – n (A \cap C) + n (A \cap B \cap C)$

Example1.

If X and Y are two sets such that $X \cup Y$ has 50 elements, X has 28 elements and Y has 32 elements, how many elements does $X \cap Y$ have ?

Solution: $n (X \cup Y) = 50$, $n (X) = 28$, $n (Y) = 32$, $n (X \cap Y) = ?$

Using formula, $n (X \cup Y) = n (X) + n (Y) – n (X \cap Y)$,

Therefore, $n (X \cap Y) = n (X) + n (Y) – n (X \cup Y) = 28 + 32 – 50 = 10$

Example2:

In a class of 35 students, 24 like to play cricket and 16 like to play football. Also, each student likes to play at least one of the two games. How many students like to play both cricket and football?

Solution :

Let X be the set of students who like to play cricket and Y be the set of students who like to play football. Then $X \cup Y$ is the set of students who like to play at least one game, and $X \cap Y$ is the set of students who like to play both games.

Given $n(X) = 24$, $n(Y) = 16$, $n(X \cup Y) = 35$, $n(X \cap Y) = ?$

Using the formula $n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$, we get $35 = 24 + 16 - n(X \cap Y)$

Thus, $n(X \cap Y) = 5$ i.e., 5 students like to play both games.

****