Class XI - MATHEMATICS

Chapter 1- SETS

Hand out of Module-1/2

Learning Outcome:

In this module we are going to learn

- Definition of Set
- Representation of set
- Empty Set
- Finite and Infinite sets
- Equal sets
- Subsets

Introduction

In everyday life, we often speak of collections of objects of a particular kind, such as, a pack of cards, a crowd of people, a cricket team, etc. In mathematics also, we come across collections.

For example, collection of natural numbers, even numbers, odd numbers, prime numbers, points etc. More specially, we examine the following collections:

- The rivers of India
- The vowels in the English alphabet, namely, a, e, i, o, u
- Various kinds of triangles
- The solution of the equation: $x^2 - 7x + 12 = 0$ .That is x=3 and x=4
- Prime numbers less than 10

We note that each of the above example is a well-defined collection of objects

For example, we can say that the river Nile does not belong to the collection of rivers of India. On the other hand, the river Ganga does belong to this collection.

Again the collection of five most renowned mathematicians of the world is not well-defined, because the criterion for determining a mathematician as most renowned may vary from person to person. Thus, it is not a well-defined collection.

Definition: **A set is a well-defined collection of objects.**

Note: i) Objects, elements and members of a set are synonymous terms.

ii) Sets are usually denoted by capital letters A, B, C, X, Y, Z, etc.

iii) The elements of a set are represented by small letters a, b, c, x, y, z, etc.

iv) If a is an element of a set A, we say that " a belongs to A"

The Greek symbol ∈ (epsilon) is used to denote the phrase 'belongs to'. Thus, we write $a \in A$.

If 'b' is not an element of a set A, we write $b \notin A$ and read "b does not belong to A".

Example: In the set V of vowels in the English alphabet, $a \in V$ but $b \notin V$.

Given below are few more examples of sets used particularly in mathematics, viz.

N : the set of all natural numbers

Z : the set of all integers

Q : the set of all rational numbers

R : the set of real numbers

Z+ : the set of positive integers

Q+ : the set of positive rational numbers, and

R+ : the set of positive real numbers.

Representation of Sets

There are two methods of representing a set.

1) **Roster or Tabular form** :

In the roster form, we list all the members of the set within braces { } and separate by commas.

Examples 1.

The set of all even positive integers less than 7 is described in roster form as {2, 4, 6 }.

Example 2.

The set of all natural numbers which divide 42 is {1, 2, 3, 6, 7, 14, 21, 42 }.

Note: i) In roster form, the order in which the elements are listed is immaterial.

Thus, the sets {1, 3, 6, 7} and {7,3,1,6 } are same.

ii) It may be noted that while writing the set in roster form an element is not generally repeated, i.e., all the elements are taken as distinct.

Example:

the set of letters forming the word 'SCHOOL' is { S, C, H, O, L}

2) **Set-builder form** :

In the set-builder form, we list the property or properties satisfied by all the elements of the sets.

Example 1:

In the set {a, e, i, o, u}, all the elements possess a common property, namely, each of them is a vowel in the English alphabet, and no other letter possess this property.

Denoting this set by V, we write V = {x : x is a vowel in English alphabet}

Example 2:

Write the solution set of the equation $x^2 – x - 12 = 0$ in a) roster form b)set builder form

Solution : The given equation can be written as

$(x + 3) (x – 4) = 0$, i. e., $x = – 3, 4$

Let A be the solution set of the given equation. Then,

a) The solution in roster form is A = {-3, 4}.

b) The solution in set builder for is A = {x : x is the solution set of the equation $x^2 – x - 12 = 0$ }

Types of Sets :

1. Empty Set: A set which does not contain any element is called an empty set or the void set or null set and it is denoted by { } or Φ.

Example:

Let A = { x : x is a student presently studying in both Classes X and XI }

We observe that a student cannot study simultaneously in both Classes X and XI.

Thus, the set A contains no element at all. Therefore A = { } or Φ

2. Finite and infinite Set: A set which is empty or consists of a definite number of elements is called a finite set otherwise, the set is called infinite.

Example 1:

Let W be the set of the days of the week. Then W is finite.

Example 2:

Let G be the set of points on a line. Then G is infinite.

Note: It is not possible to write all the elements of an infinite set within braces { } because the numbers of elements of such a set is not finite. So, we represent an infinite set in the roster form by writing a few elements which clearly indicate the structure of the set followed (or preceded) by three dots.

Examples.

{1, 2, 3 . . .} is the set of natural numbers

{1, 3, 5, 7, . . .} is the set of odd natural numbers,

{. . .,–3, –2, –1, 0,1, 2 ,3, . . .} is the set of integers. All these sets are infinite.

Note: All infinite sets cannot be described in the roster form.

Example.

The set of real numbers cannot be described in roster form because, the elements of this set do not follow any particular pattern.

Note: A set consists of a single element, is called a singleton set. Thus, {a} is a singleton set.

3. Equal Sets : Two sets A and B are said to be equal if they have exactly the same elements and we write A = B. Otherwise, the sets are said to be unequal and we write A ≠ B.

Example 1 :

Let A = {1, 2, 3, 4} and B = {3, 1, 4, 2}. Then A = B.

Example 2:

Let A be the set of prime numbers less than 6 and P the set of prime factors of 30.

2, 3 and 5 are the only prime factors of 30 and also these are less than 6.

Therefore A ={2,3,5} and P ={2, 3, 5}.Thus A =P

Note: A set does not change if one or more elements of the set are repeated.

Example. The sets A = {1, 2, 3} and B = {2, 2, 1, 3, 3} are equal, since each element of A is in B and vice-versa. That is why we generally do not repeat any element in describing a set.

Example

Find the pairs of equal sets, if any, give reasons:

A = {0}, B = {x : x > 15 and x < 5}, C = {x : x – 5 = 0 }, D = {x: x^2 = 25},

E = {x : x is an integral positive root of the equation $x^2 - 2x - 15 = 0$}.

Solution Since 0 ∈ A and 0 does not belong to any of the sets B, C, D and E, it follows that,

A ≠ B, A ≠ C, A ≠ D, A ≠ E.

Since B = Φ but none of the other sets are empty. Therefore B ≠ C, B ≠ D and B ≠ E.

Also C = {5} , D = {–5, 5} and E = {5}, Therefore C = E.

Further –5 ∈ D, hence C ≠ D. and D ≠ E. Thus, the only pair of equal sets is C and E.

4. Subset: A set A is said to be a subset of a set B if every element of A is also an element of B.

In other words, A ⊂ B if whenever a ∈ A, then a ∈ B. That is A ⊂ B if a ∈ A ⇒ a ∈ B

Example 1:

The set **Q** of rational numbers is a subset of the set **R** of real numbers, and we write **Q** ⊂ R.

Example 2:

If A is the set of all divisors of 56 and B the set of all prime divisors of 56,

then B is a subset of A and we write B ⊂ A.

Example 3:

Let A = {1, 3, 5} and B ={x : x is an odd natural number less than 6}.

Then A⊂ B and B ⊂ A and hence A = B.

Example 4:

Let A = { a, e, i, o, u} and B = { a, b, c, d}. Then A is not a subset of B, also B is not a subset of A.

Note: Let A and B be two sets. If A ⊂ B and A ≠ B , then

A is called a proper subset of B and B is called superset of A.

Example

A = {1, 2, 3} is a proper subset of B = {1, 2, 3, 4}.

Some Subsets of set of real numbers :

- The set of natural numbers **N** = {1, 2, 3, 4, 5, . . .}
- The set of integers **Z** = {. . ., –3, –2, –1, 0, 1, 2, 3, . . .}
- The set of rational numbers **Q** = { x : x = $\frac{p}{q}$, p, q ∈ **Z** and q ≠ 0}
- **T** = {x : x ∈ R and x ∉ **Q**}, i.e., all real numbers that are not rational.

  Members of **T** include $\sqrt{2}$, $\sqrt{5}$, and π.

  Here, N ⊂ Z ⊂ Q, Q ⊂ R, T ⊂ R, N ⊄T.

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