CLASS XI
SUBJECT : MATHEMATICS
LESSON: STRAIGHT LINES
MODULE – 3/3

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- In this module we will study about

- General form of equation of straight lines
- Conversion of general equation into different forms
- Distance of a point from a line
- Distance between two parallel lines
- Example problems
- Problems for practice

- **General Form of a line:**
- We know that general equation of first degree in two variables is $Ax + By + C = 0$, where A, B and C are constants.
- Also graph of he above equation is always a straight line .
- Hence we get the general equation of a straight line will be of the form $Ax + By + C = 0$.

- **Different forms of $Ax + By + C = 0$**
- The general equation of the straight line can be reduced into various form of the equation of the line.

a. Slope-interept form:

If B ≠ 0 , then Ax + By + C = 0 can be written as

$y = -\frac{A}{B}x - \frac{C}{B}$, this is of the form y = mx + c

where $m = -\frac{A}{B} = -\frac{Coefficient\ of\ x}{Coefficient\ of\ y}$ and $C = -\frac{C}{B} = -\frac{Cconstant\ term}{coefficient\ of\ y}$

b. Intercept form:

If C ≠ 0 , then Ax + By + C = 0 can be written as

Ax + By = -C

$\Rightarrow \frac{x}{-\frac{C}{A}} + \frac{y}{-\frac{C}{B}} = 1$, this is of the form $\frac{x}{a} + \frac{y}{b} = 1$,

where $a = -\frac{C}{A}$ and $b = -\frac{C}{B}$

c. Normal form:

Let x cosw + y sinw = p-----------(i)

be the normal form of the line represented by the line

Ax + By + C = 0 (or) Ax + By = - C .----------(ii)

If both the equation are same then,

$$\frac{A}{cosw} = \frac{B}{sinw} = -\frac{C}{p}$$

This gives cosw = $-\frac{Ap}{C}$ and sinw = $-\frac{Bp}{C}$

Now, $\sin^2 w + \cos^2 w = (-\frac{Ap}{C})^2 + (-\frac{Bp}{C})^2$

$$\Rightarrow 1 = \frac{p^2(A^2+B^2)}{C^2} \qquad \Rightarrow p^2 = \frac{C^2}{(A^2+B^2)}$$

$$\Rightarrow p = \pm\frac{C}{\sqrt{A^2+B^2}}$$

Therefore , cos w = $\pm\frac{A}{\sqrt{A^2+B^2}}$ and

sin w = $\pm\frac{B}{\sqrt{A^2+B^2}}$

Thus, the normal form of the equation

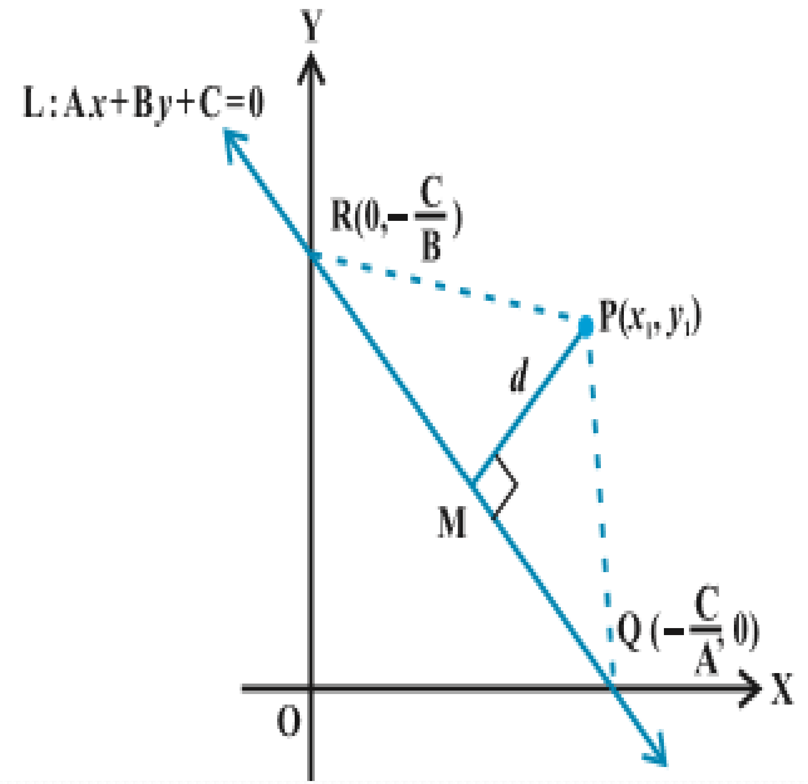
Ax + By + C = 0 becomes

x cosw + y sinw = p, where cosw, sinw and p

can be found from the above

Distance of a point from a line

Let L be the line Ax + By + C =0 , whose distance from the point P(x_1,y_1) is 'd'.
Draw a perpendicular PM from the point p to the line. Let the line meets x and y axes at Q and R respectively.
The coordinates of Q and R are
$Q\left(-\frac{C}{A}, 0\right)$and $R\left(0, -\frac{C}{B}\right)$



Now, area of triangle PQR is given by

$$\text{Area}(\Delta PQR) = \frac{1}{2}.\text{PM.QR} = \frac{1}{2}.\text{d.QR}$$

$$\Rightarrow \text{d} = \frac{2.\text{Area}(\Delta PQR)}{QR} \quad \text{--------(1)}$$

QR = $\sqrt{\left(0+\frac{C}{A}\right)^2+\left(\frac{C}{B}-0\right)^2} = \frac{C}{AB}\sqrt{(A)^2+(B)^2}$ -------------(2)

Also, Area(ΔPQR) = $\frac{1}{2}\left|x_1\left(0+\frac{C}{B}\right)+\left(-\frac{C}{A}\right)\left(-\frac{C}{B}-y_1\right)+0(y_1-0)\right|$

$= \frac{1}{2}\left|x_1\left(\frac{C}{B}\right)+y_1\left(\frac{C}{A}\right)+\frac{C^2}{AB}\right| = \frac{1}{2}\cdot\frac{C}{AB}|Ax_1+By_1+C|$

⇒ 2 Area(ΔPQR) = $\frac{C}{AB}|Ax_1+By_1+C|$-----------(3)

Substituting the value of (2) and (3) in (1) we get ,

d = $\frac{2.\text{Area}(\Delta PQR)}{QR}$ ⇒ d = $\frac{\frac{C}{AB}|Ax_1+By_1+C|}{\frac{C}{AB}\sqrt{(A)^2+(B)^2}} = \frac{|Ax_1+By_1+C|}{\sqrt{(A)^2+(B)^2}}$

Thus , the distance from a point P(x_1,y_1) to the line Ax + By + C =0 is

d = $\mathbf{\frac{|Ax_1+By_1+C|}{\sqrt{(A)^2+(B)^2}}}$

Note: If the point P is the origin the distance becomes,

$$\mathbf{d} = \frac{|A(0)+B(0)+C|}{\sqrt{(A)^2+(B)^2}} = \frac{|C|}{\sqrt{(A)^2+(B)^2}}$$

Distance between two parallel lines

Let $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ be two parallel lines. (Two parallel lines differ only in their constant terms as their slopes are equal)

The distance between the parallel lines is

$$\mathbf{d} = \frac{|C1-C2|}{\sqrt{(A)^2+(B)^2}}$$

Example problems

Example-1.

Find the distance between the parallel lines

$$3x - 4y + 7 = 0 \text{ and } 3x - 4y + 5 = 0$$

Solution: Here A = 3, B = -4 , C_1 = 7 and C_2 = 5.

Distance between the lines is

$$\mathbf{d} = \frac{|C1 - C2|}{\sqrt{(A)^2 + (B)^2}} = \frac{|7-5|}{\sqrt{(3)^2 + (-4)^2}} = \frac{2}{5}.$$

Example -2

Find the equation of the line perpendicular to the line $x - 2y + 3 = 0$ and passing through the point (1,-2).

Solution:

Given line is $x - 2y + 3 = 0$

Slope of the line M = $-\frac{coeff.of\ x}{coeff.of\ y} = -\frac{1}{-2} = \frac{1}{2}$

Since the line is perpendicular to the given line,

slope of the required line is m = -2

So equation of the line is in slope point form

$y-y_1 = m(x-x_1) \Rightarrow y-(-2) = -2(x-1)$

$\Rightarrow y + 2 = -2x + 2$

$\Rightarrow 2x + y = 0$

Which is the required line

Example -3:

Find the distance of the line $4x - y = 0$ from the point P(4,1) measured along the line which is making an angle of 135^0 with the positive x-axis

Solution:

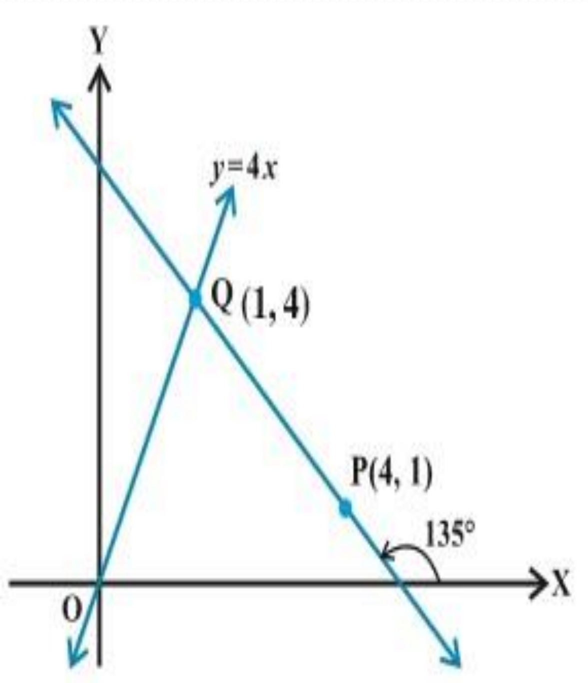
Given line is $4x-y = 0$ ----------(1)

Equation of the line which makes an angle 135^0 with x –axis and passing through the point (4,1) is

$y - 1 = \tan 135^0 (x-4)$.

$\Rightarrow y - 1 = -1(x - 4)$

$\Rightarrow x + y - 5 = 0$ ---------------------(2)



Solving (1) and (2) we get the point Q as (1,4)

Given point P is (4,1)

So from the problem, required distance

$PQ = \sqrt{(1-4)^2 + (4-1)^2}$

$= \sqrt{9+9}$

$= 3\sqrt{2}$ units

Example -4:

Find the value of k so that the line 2x + ky – 9 = 0 may be

(i) Parallel to 3x – 4y + 7 = 0

(ii) Perpendicular to 3y + 2x – 1 = 0

Solution:

Given line is 2x + ky – 9 = 0

Slope of the line is M = $-\frac{2}{k}$ -----------(1)

(i). Slope of the line 3x – 4y + 7 = 0 is m = $\frac{3}{4}$ -----------(2)

Since the two lines are parallel M = m

$\Rightarrow -\frac{2}{k} = \frac{3}{4} \quad \Rightarrow k = -\frac{8}{3}$

(ii). Slope of the line 3x – 2y – 1 = 0 is $\frac{-3}{-2} = \frac{3}{2}$

Slope perpendicular to the above line m_1= - $\frac{2}{3}$ ----------(3)

Since the given line is perpendicular to (3)

$\mathbf{M.m_1 = -1} \Rightarrow (-\frac{2}{k}).(-\frac{2}{3}) = -1$

$\Rightarrow 4 = -3k \Rightarrow k = -\frac{4}{3}$

Example-5:

If p is the length of perpendicular from the origin to the line $\frac{x}{a}+\frac{y}{b}=1$ which makes intercepts 'a' and 'b' with the axes, prove that $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$.

Solution:

Given line is $\frac{x}{a}+\frac{y}{b}-1=0$.------------(i)

Since 'p'is the length of perpendicular from the origin (o,o) to the line (i),

$$p=\frac{|-1|}{\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2}} \Rightarrow p^2=\frac{1}{\frac{1}{a^2}+\frac{1}{b^2}}$$

$\Rightarrow \frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$, as required.

Example – 6

A variable line passes through a fixed point P. The algebraic sum of the perpendiculars dawn from the point A(2,0),B(0,2) and C (1,1) on the line is zero. Find the coordinate of the point P.

Solution:

Let the slope of the line is ‘m’ and the fixed point P is (x_1,y_1).

So, the equation of the line is $y - y_1 = m(x - x_1)$ --------(1)

The perpendicular distance from A(2,0) to line (1) is

$$d_1 = \left|\frac{0-y_1-m(2-x_1)}{\sqrt{1+m^2}}\right|$$

Similarly,

The perpendicular distance from B(0,2) to line (1) is

$$d_2 = \left|\frac{2-y_1-m(0-x_1)}{\sqrt{1+m^2}}\right| \quad \text{and}$$

distance from C(1,1) to the line (1) is

$$d_3 = \left|\frac{1-y_1-m(1-x_1)}{\sqrt{1+m^2}}\right|$$

According to the problem , $d_1 + d_2 + d_3 = 0$

$$\Rightarrow \quad \left|\frac{-y_1-2m+mx_1+2-y_1+mx_1+1-y_1-m+mx_1)}{\sqrt{1+m^2}}\right| = 0$$

$\Rightarrow -3y_1 - 3m + 3mx_1 + 3 = 0$

$\Rightarrow -y_1 - m + mx_1 + 1 = 0$

Since the point (1,1,) lies on this , the point P is (1,1).

Problem for Practice:

All problems from Exercise10.3 and miscellaneous exercise from class XI NCERT mathematics Text book.