CLASS XI

SUBJECT : MATHEMATICS

LESSON: STRAIGHT LINES

MODULE – 1/3

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STRAIGHT LINES

In this module we will study about

- Some Important formula from previous classes
- Slope of a straight line

i. If the angle made by the line with x-axis is known

ii. When the coordinates of any two points on the line is given

- Angle between two lines
- Conditions for parallel and perpendicular lines
- Collinearity of three points
- Some example problems
- Problems for practice

Recall of important formula

1. Distance between two points $P(x_1,y_1)$ and $Q(x_2,y_2)$ is

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

2. Section formula

The coordinates of a point dividing the line segment joining the point (x_1, y_1) and (x_2, y_2) internally in the ratio m: n are

$$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

3. In particular when m = n, the coordinate of the midpoint of the line segment joining the points (x_1,y_1) and (x_2,y_2) are

$$\left(\frac{x_2+x_1}{2},\ \frac{y_2+y_1}{2}\right)$$

4. Area of the triangle whose vertices are (x_1,y_1) ,(x_2,y_2) and (x_3,y_3) is

$$\frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$$

Remark: If the area of the triangle ABC is zero, then the three points A,B and C lie on a line, i.e, they are collinear

i.e, $\frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right| = 0$

$\Rightarrow \left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right| = 0$

Slope of a straight line

Any line (l) in the coordinate plane forms two angles with the x-axis which are supplementary. If **θ** be the angle made by the line 'l' with the positive direction of x- axis and measured anti-clockwise is called the inclination of the line and obviously $o \leq \theta \leq 180$.

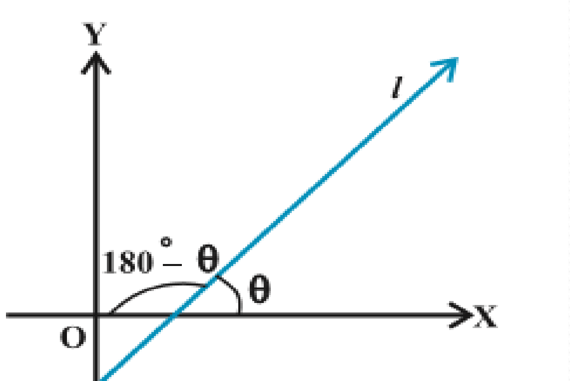


Fig 10.2

If **θ** is the inclination of the line with x-axis then ‘**tan θ**’ is called the **slope** (or) gradient of the line and it is denoted as ‘**m**’

Thus ,slope of a line m = tan **θ**

Note: If = 0°, then the line is parallel to x-axis and if **θ** = 90° , then the line is perpendicular to x-axis (or) Parallel to y-axis.

Slope of a line when the any two points of the line is given

Consider the diagram, Let $P(x_1,y_1)$ and $Q(x_2,y_2)$ on the line 'l' and let θ be the inclination of the line with x-axis.

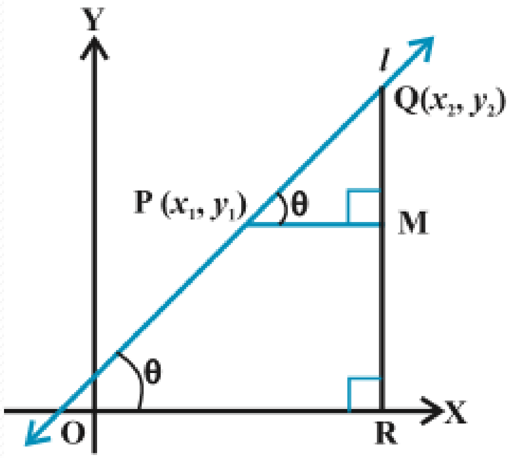


Fig 10. 3 (i)

Draw the perpendiculars QR to x-axis and PM to QR.

From the diagram, $\lfloor MPQ = \theta$.

Hence , Slope of the line l = m = tan θ

$$= \frac{MQ}{MP} = \frac{y_2 - y_1}{x_2 - x_1}$$

Simillarly, when θ is an obtuse angle we can find the slope as the same way.

Angle between two straight lines (in terms of slop)

Let L_1 and L_2 be two non-vertical lines with slopes m_1 and m_2. If $\alpha 1$ *and* $\alpha 2$ are the inclinations of the lines with x-axis then $m_1 = \tan \alpha 1$ and $m_2 = \tan \alpha 2$

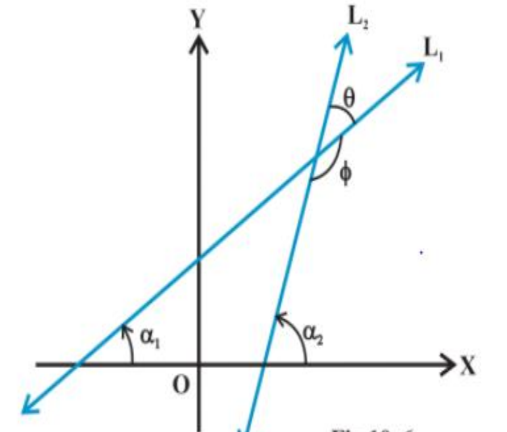


Fig 10. 6

We know that, when two lines intersect each other, they make a pair of vertically opposite angles such that their sum is 180^o .

If θ *and* φ be the adjacent angles then $\theta = \alpha 2 - \alpha 1$ and $\varphi = 180 - \theta$

Now, $\tan\theta = \tan(\alpha 2 - \alpha 1)$

$$= \frac{\tan\alpha_2 - \tan\alpha_1}{1+\tan\alpha_1 . \tan\alpha_2} = \frac{m_2 - m_1}{1+m_1 . m_2} \text{ as } 1+ m_1 m_2 \neq 0 \text{---------(i)}$$

Also, $\tan\varphi = \tan(180 - \theta)$

$$= -\tan\theta = -\frac{m_2 - m_1}{1 + m_1.m_2}$$

i,e, $\tan\varphi = -\frac{m_2 - m_1}{1 + m_1.m_2}$ --------------------(ii)

Thus from (i) and (ii) we conclude that If tan**θ** is positive then $\tan\varphi$ is negative which means when **θ** is acute $\boldsymbol{\varphi}$ is obtuse and vice-versa.

Thus we get the formula for angle between two lines as

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \text{ as } 1 + m_1 m_2 \neq 0.$$

This will always give the acute angle between two lines. We can find the other angle (obtuse) by using $180 - \theta$

Conditions for parallel and perpendicular lines

We have, $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

Case(i): If the two lines are parallel, then the angle between the lines $\boldsymbol{\theta} = 0$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \Rightarrow \frac{m_1 - m_2}{1 + m_1 m_2} = 0$$

$$\Rightarrow m_1 - m_2 = 0$$

$$\Rightarrow m_1 = m_2$$

This is the condition for two lines to be parallel

Case(ii): If the two lines are perpendicular, then the angle between the lines $\boldsymbol{\theta} = 90$

$$\tan 90 = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \Rightarrow 1 + m_1 m_2 = 0$$

$$\Rightarrow m_1 m_2 = -1$$

This is the condition for two lines to be perpendicular to each other

<u>Collinearity of three points</u>

We know that slope of two parallel lines are always equal. If two lines have the same slope and passes through a common point , then the two lines will coincide.

Moreover, if three points are collinear then the area of the triangle will become zero.

i.e, $\frac{1}{2}\,|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| = 0$

$\Rightarrow x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$

This is the condition for three point to be collinear.

<u>Example Problems</u>

<u>Example-1.</u> If the distance between the two points (a,-2) and (5,1) is 5 units, find the value(s) of a.

Solution: The distance between (a,-2) and (5,1)

$$= \sqrt{(5-a)^2 + (1-(-2)^2}$$

According to the given problem,

$$\sqrt{(5-a)^2 + (1-(-2)^2} = 5$$

$$\Rightarrow \sqrt{(5-a)^2 + (3)^2} = 5$$

$$\Rightarrow (5-a)^2 + (3)^2 = 25$$

$$\Rightarrow (5-a)^2 = 25 - 9 = 16$$

$$\Rightarrow 5 - a = \pm 4$$

$$\Rightarrow a = 1,\ 9$$

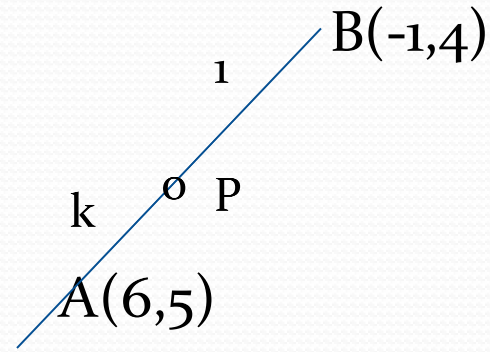
Hence, the required values of a are 1, 9

Example:2.

Find the ratio in which the point P whose abscissa is 3 divides the join of A(6,5) and B(-1,4). Hence, find the coordinates of P.

Solution: Let P divides the segment AB in the ratio k : 1

Hence by section formula,



the coordinates of P are $\left(\frac{-k+6}{k+1}, \ \frac{4k+5}{k+1}\right)$

But abscissa of point P is 3 (given)

So, $\frac{-k+6}{k+1} = 3 \quad \Rightarrow 3k + 3 = -k + 6$

$\Rightarrow 4k = 3 \Rightarrow k = \frac{3}{4}$

Hence the required ratio is $\frac{3}{4}$:1, i.e, 3:4 internally

$\therefore$ Coordinate of P are $\left(\frac{\frac{-3}{4}+6}{\frac{3}{4}+1}, \frac{4\frac{3}{4}+5}{\frac{3}{4}+1}\right) = \left(\frac{21}{7}, \frac{32}{7}\right) = \left(3, \frac{32}{7}\right)$.

Example :3-If the points A(0,4),B(1,2) and C(3,3) are three corners of a square, find

(i)The coordinate of the point at which, the diagonals intersect.

(ii) The coordinate of D, the fourth corner of the square.

Solution: Let the fourth corner D be (a , b)

Now, AB= $\sqrt{(1-0)^2+(2-4)^2} = \sqrt{1+4} = \sqrt{5}$

BC= $\sqrt{(3-1)^2+(3-2)^2} = \sqrt{4+1} = \sqrt{5}$

AC= $\sqrt{(3-0)^2+(3-4)^2} = \sqrt{9+1} = \sqrt{10}$

We note that, AB = BC and $AC^2 = AB^2 + BC^2$

i.e, 10 = 5 + 5

So ABCD is a square and BD is another diagonal

(i) Since diagonals of square bisect at the mid-point

Point of intersection = midpoint of AC = $\left(\frac{0+3}{2}, \frac{4+3}{2}\right)$

$= \left(\frac{3}{2}, \frac{7}{2}\right)$

(ii) mid point of AC = mid-point of BD

$$\left(\frac{0+3}{2}, \frac{4+3}{2}\right) = \left(\frac{a+1}{2}, \frac{b+2}{2}\right)$$

$\Rightarrow \frac{a+1}{2} = \frac{3}{2}$ and $\frac{b+2}{2} = \frac{7}{2}$

$\Rightarrow$a = 2 and b = 5

So the fourth corner is (2,5)

Example-4: Find the area of the triangle whose vertices are (10,-6),(2,5) and (-1,3)

Solution: The area of the triangle whose vertices are (10,-6),(2,5) and (-1,3)

$$= \frac{1}{2}|10(5-3)+2(3+6)+(-1)(-6-5)|$$

$$= \frac{1}{2}|20+18+11| = \frac{49}{2} \text{ sq.units.}$$

Example 5: If the vertices of the triangle are (1,k),(4,-3) and (-9,7) and its area is 15 sq. units , find the value(s) of k.

Solution: Area of the triangle formed by the given points

$$= \frac{1}{2}|1(-3-7)+4(7-k)+(-9)(k+3)|$$

$$= \frac{1}{2}|-10+28-4k-9k-27|$$

$$= \frac{1}{2}|-9-13k|$$

As per the problem, $\frac{1}{2}|-9-13k| = \pm 15$

$\Rightarrow 9 + 13k = \pm 30 \Rightarrow 13k = -39 , 21$

$\Rightarrow k = -3 , 21/13$

Example 6: For what values of x are the points (1,5),(x,1) and (4,11) are collinear?

Solution: Since the points are collinear,

Area of the triangle formed by the points = 0

i.e, $\frac{1}{2}|1(1-11)+x(11-5)+(4)(5-1)| = 0$

$\Rightarrow |-10+6x+16| = 0$

$\Rightarrow 6x + 6 = 0$

$\Rightarrow x = -1$

Hence, the required value of x is -1

Example 7: Find the angle between the lines joining the points (-1,2) ,(3,-5) and (-2,3),(5,0).

Solution:

Slope of the line joining (-1,2) ,(3,-5) is

$$m_1 = \frac{-5-2}{3+1} = \frac{-7}{4}$$

Slope of the line joining (-2,3),(5,0)is

$$m_2 = \frac{0-3}{5+2} = \frac{-3}{7}$$

Let θ be the angle between the given lines, then $\tan\theta = \left|\frac{m_1-m_2}{1+m_1.m_2}\right|$

$$= \left|\frac{-\frac{7}{4}-(\frac{-3}{7})}{1+(-\frac{7}{4})(\frac{-3}{7})}\right| = \left|\frac{-\frac{37}{28}}{\frac{7}{4}}\right| = \frac{37}{49}$$

Hence the acute angle between the lines is given by $\tan\theta = \frac{37}{49}$

Problems for Practice

Exercise 10.1 complete from NCERT text book for class XI Mathematics