SYSTEM OF PARTICLES AND ROTATIONAL MOTION

SUBJECT: PHYSICS

CLASS: XI

MODULE 1 OF 3

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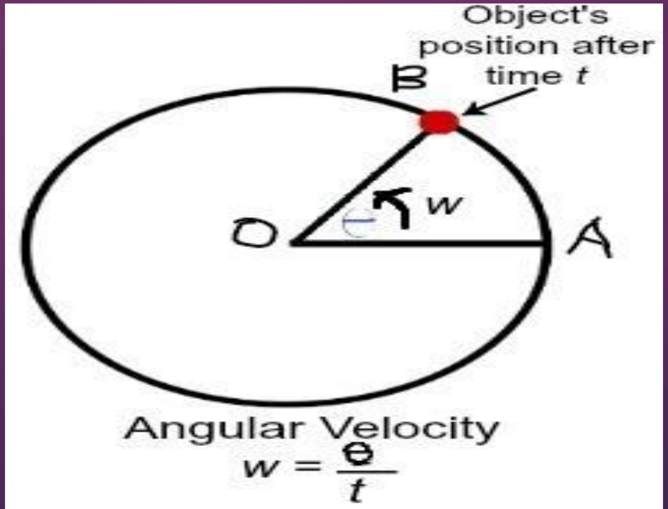
System of Particles and Rotational Motion

In this module, we will discuss the meaning of angular velocity, angular acceleration and the equations of motion for rotational motion.

Angular Velocity (ω)

Angular velocity is the angle described by a rotating body per unit time.

Let the angle BOA described by the particle in time t is ϴ radian. The magnitude of angular velocity will be,



$$\omega = \frac{\theta}{t}$$

If the particle describes one complete revolution then,

$$\omega = \frac{2\pi}{t}$$

Here, $\mathrm{t} = T$. Therefore,

$$\omega = \frac{2\pi}{T}$$

If the particle describes n revolutions in one second then,

$$\mathrm{n} = \frac{1}{T}$$

Hence, $\omega = 2\pi\mathrm{n}$

The angular velocity is measured in radian $second^{-1}$.

Its dimensional formula is $[T^{-1}]$.

Uniform Angular Velocity

If the particle describes equal angles in equal intervals of time then the angular velocity is said to be uniform.

$$\omega \text{ (uniform angular velocity)} = \frac{\theta \text{ (equal angles described)}}{t \text{ (equal intervals of time)}}$$

Angular acceleration (α)

Angular acceleration is the rate of change of angular velocity with time.

It is a vector quantity and denoted by α.

$$\alpha = \frac{\text{change in angular velocity}}{\text{time}}$$

$$\alpha = \frac{\omega_2 - \omega_1}{t - 0} = \frac{2\pi n_2 - 2\pi n_1}{t - 0} = \frac{2\pi(n_2 - n_1)}{t}$$

Here, n_2 and n_1 are the number of revolutions made by the particle in one second.

Angular acceleration is measured in radian sec^{-2}.

Its dimensional formula is $[T^{-2}]$.

Equations of Motion for Rotational Motion

For translatory motion, the equations of motion are as follows,

$$v = u + at$$
$$s = ut + \frac{1}{2}at^2$$
$$v^2 = u^2 + 2as$$

For rotational motion, the equations of motion are analogous to that for translatory motion. These are,

$$\omega = \omega_0 + \alpha t$$
$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$
$$\omega^2 = \omega_0{}^2 + 2\alpha\theta$$

Mathematical Derivation of the First Equation of Motion for Rotational Motion

Angular acceleration $\alpha = \frac{\text{change in angular velocity}}{\text{time}}$

Let ω_0 be the initial angular velocity and ω be the final angular velocity after time t.

$$\alpha = \frac{\omega - \omega_0}{t - 0} = \frac{\omega - \omega_0}{t}$$

$$\alpha t = \omega - \omega_0$$

$$\omega = \omega_0 + \alpha t$$

This equation is known as the first equation of motion for rotational motion.

It describes the relation between initial angular velocity, final angular velocity and angular acceleration.

Mathematical Derivation of the Second Equation of Motion for Rotational Motion

For translatory motion, the displacement is given as,

$$s = average\ velocity \times time\ interval$$

Similarly for rotational motion, the angular displacement is given as,

$$\theta = average\ angular\ velocity \times time\ interval$$

$$\theta = \frac{(\omega + \omega_0)}{2} \times t$$

From the first equation of motion for rotational motion we have,

$$\omega = \omega_0 + \alpha t$$

Thus on substituting we get,

$$\theta = \frac{(\omega_0 + \alpha t + \omega_0)}{2} \times t$$

$$\theta = \frac{(2\omega_0 + \alpha t)}{2} \times t$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

This equation is known as the second equation of motion for rotational motion.

It describes the relation between initial angular displacement, angular velocity, angular acceleration and time taken.

Mathematical Derivation of the Third Equation of Motion for Rotational Motion

For rotational motion, the angular displacement is given as,

$$\theta = average\ angular\ velocity \times time\ interval$$

$$\theta = \frac{(\omega + \omega_0)}{2} \times t$$

We know that,

$$\alpha = \frac{\omega - \omega_0}{t}$$

Hence,

$$t = \frac{\omega - \omega_0}{\alpha}$$

Therefore,

$$\theta = \frac{(\omega + \omega_0)(\omega - \omega_0)}{2\alpha}$$

$$\theta = \frac{\omega^2 - {\omega_0}^2}{2\alpha}$$

$$2\alpha\theta = \omega^2 - {\omega_0}^2$$

$$\omega^2 = {\omega_0}^2 + 2\alpha\theta$$

This equation is known as the third equation of motion for rotational motion.

It describes the relation between initial angular velocity, final angular velocity, angular acceleration and angular displacement.

We shall now discuss some problems based on these equations of motion.

Problem 1

A wheel starts rotating at 10 rad s^{-1} and attains an angular velocity of 100 rad s^{-1} in 15 s. What is the angular acceleration in rad sec^{-2}?

(a) 10 rad s^{-2} (b) $\frac{110}{15}$ rad s^{-2} (c) $\frac{100}{15}$ rad s^{-2} (d) 6 rad s^{-2}

Solution: $\alpha = \frac{\omega - \omega_0}{t} = \frac{100-10}{15} = \frac{90}{15} = 6$ rad s^{-2}

Problem 2

A wheel starts rotating from rest and attains an angular velocity of 60 rad s^{-1} in 5 s. The total angular displacement in radian will be

(a) 60 rad (b) 80 rad (c) 100 rad (d) 150 rad

Solution: $\omega_0 = 0$, $\omega = 60$ rad s^{-1}, $t = 5$ s

Angular acceleration, $\alpha = \frac{\omega - \omega_0}{t} = \frac{60-0}{5} = 12$ rad s^{-2}

Angular displacement, $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\theta = 0 \times 5 + \frac{1}{2} \times 12 \times (5)^2 = 0 + \frac{300}{2} = 150$ rad

Problem 3

A wheel having a diameter of 3 m starts from rest and accelerates uniformly to an angular velocity of 210 rpm in 5 s. The angular acceleration of the wheel is

(a) 1.4π rad s^{-2} (b) 3.3π rad s^{-2} (c) 2.2π rad s^{-2} (d) 1.1π rad s^{-2}

Solution: $\omega_0 = 0$, t = 5 s

$$n = 210\ \text{rpm} = \frac{210}{60} = \frac{7}{2}\ rps$$

But,

$$\omega = 2\pi n = 2 \times \pi \times \frac{7}{2} = 7\pi$$

Thus angular acceleration is given as,

$$\alpha = \frac{\text{change in angular velocity}}{\text{time}} = \frac{7\pi - 0}{5} = 1.4\pi\ rad\ s^{-2}$$

THANK YOU