ATOMIC ENERGY EDUCATION SOCIETY, MUMBAI
CLASS: XII (MATHEMATICS)

CHAPTER - 09
TOPIC: DIFFERENTIAL EQUATIONS
HANDOUT: MODULE 3/3

Methods of Solving First Order, First Degree Differential Equations:-

(3) Linear differential equations:

A differential equation of the from

$$\frac{dy}{dx} + P\,y = Q$$

where, P and Q are constants or functions of *x* only, is known as a first order linear differential equation.

Examples: (i) $\frac{dy}{dx} + 2y = e^{5x}$

(ii) $\frac{dy}{dx} + \frac{1}{x}y = \cos x$

(iii) $\frac{dy}{dx} + y \sec x = 7$

Method of solving the first order linear differential equation:

(i) Write the given differential equation in the form

$$\frac{dy}{dx} + P\,y = Q$$

(ii) Find the Integrating Factor (I.F) = $e^{\int p dx}$

(iii) Write the solution formula of the given differential equation as:

$$y.\,(\text{I.F}) = \int Q.(I.F)dx + C$$

which gives the general solution after integration on right hand side.

Example: - (i) Find the general solution of the differential equation

$$\frac{dy}{dx} + 3y = e^{-2x}$$

Clearly this DE is in linear form, $\frac{dy}{dx} + P\,y = Q$

P = 3, Q = e^{-2x}

$$\text{I. F.} = e^{\int p dx}$$
$$= e^{\int 3dx}$$
$$= e^{3x}$$

Solution formula,

$$\text{y. (I.F)} = \int Q.(I.F)dx$$
$$\text{y. } e^{3x} = \int e^{-2x}.e^{3x}dx$$
$$= \int e^{x}dx$$
$$= e^{x} + \text{C}$$
$$\text{y. } e^{3x} = e^{x} + \text{C}$$

which is a general solution of the given DE.

Example: - (ii) Find the general solution of the differential equation

$$Cos^2x.\frac{\text{dy}}{\text{dx}} + \text{y} = \tan \text{x}, \ \left(0 \leq x < \frac{\pi}{2}\right)$$

The given DE is $Cos^2x.\frac{\text{dy}}{\text{dx}} + \text{y} = \tan \text{x},$

Divide both the sides by Cos^2x,

$$\frac{\text{dy}}{\text{dx}} + \frac{y}{Cos^2x} = \frac{\tan \text{x}}{Cos^2x},$$
$$\frac{\text{dy}}{\text{dx}} + \text{y Sec}^2\text{x} = \text{Sec}^2\text{x}.\tan \text{x},$$

Which is a linear DE of the form $\frac{\text{dy}}{\text{dx}} + \text{P y} = \text{Q}$

$$\text{P} = \text{Sec}^2\text{x}, \quad \text{Q} = \text{Sec}^2\text{x}.\tan \text{x},$$
$$\text{I. F.} = e^{\int p dx}$$
$$= e^{\int \text{Sec}^2\text{x}\, dx}$$
$$= e^{tanx}$$

Solution formula,

$$\text{y. (I.F)} = \int Q.(I.F)dx$$
$$\text{y. } e^{tanx} = \int \text{Sec}^2\text{x}.\tan \text{x} \quad .e^{tanx}\, dx$$

Put tan x = t

$Sec^2 x\, dx = dt$

$y.\, e^{tanx} = \int t.\, e^t dt$

Integrating by part

$y.\, e^{tanx} = t.\, e^t - \int e^t dt$

$= t.\, e^t - e^t + C$

$y.\, e^{tanx} = \tan x\, .\, e^{\tan x} - e^{\tan x} + C$

$y = (\tan x - 1) + C\, e^{-tanx}$

which is a general solution of the given DE.