

ATOMIC ENERGY EDUCATION SOCIETY, MUMBAI
CLASS: XII (MATHEMATICS)

CHAPTER - 09
TOPIC: DIFFERENTIAL EQUATIONS
HANDOUT: MODULE 1/3

BASIC CONCEPTS:-

INDEPENDENT & DEPENDENT VARIABLES:

If $y = f(x)$, then x is independent variable & y is dependent variable.

DIFFERENTIAL EQUATION:

An equation involving derivative (derivatives) of the dependent variable with respect to independent variable (variables) is called a differential equation.

Example:- (i) $\frac{dy}{dx} + 3xy = \text{Cos}x$

(ii) $\frac{d^2y}{dx^2} + x\frac{dy}{dx} = e^y$

(iii) $\frac{d^3y}{dx^3} + \text{Sin}\left(\frac{dy}{dx}\right) = x^2 \log x$

ORDINARY DIFFERENTIAL EQUATION:

A differential equation involving derivatives of the dependent variable with respect to **only one** independent variable is called an ordinary differential equation.

Example:- (i) $y\left(\frac{dy}{dx}\right)^3 - 5 \tan y = \text{Cos}x$

(ii) $\left(\frac{d^2y}{dx^2}\right)^4 + x\left(\frac{dy}{dx}\right)^5 = e^y$

(iii) $\left(\frac{d^3y}{dx^3}\right)^{\frac{3}{2}} - 7y^3\left(\frac{dy}{dx}\right) = 3x^2$

Of course, there are differential equations involving derivatives with respect to more than one independent variables, called partial differential equations but at this stage we shall confine ourselves to the study of ordinary differential equations only.

We will use the term ‘differential equation’ for ‘ordinary differential equation’.

ORDER OF A DIFFERENTIAL EQUATION:

Order of a differential equation is the order of the highest order derivative occurring in the differential equation.

Example:- (i) $\left(\frac{d^2y}{dx^2}\right)^4 + x\left(\frac{dy}{dx}\right)^5 = e^y$ Order = 2

(ii) $\frac{d^3y}{dx^3} + \text{Sin}\left(\frac{dy}{dx}\right) = x^2 \log x$ Order = 3

DEGREE OF A DIFFERENTIAL EQUATION:

Degree of a differential equation is defined if it is a polynomial equation in its derivatives.

Degree of a differential equation is the highest power (positive integer only) of the highest order derivative in it.

Example:- (i) $\left(\frac{d^2y}{dx^2}\right)^4 + x\left(\frac{dy}{dx}\right)^5 = e^y$ Degree = 4

(ii) $\frac{d^3y}{dx^3} + \text{Sin}\left(\frac{dy}{dx}\right) = x^2 \log x$ Degree = not defined

SOLUTION OF DIFFERENTIAL EQUATION:

A relation between involved variables, which satisfy the given differential equation is called its solution.

GENERAL SOLUTION:

The solution which contains as many arbitrary constants as the order of the differential equation is called the general solution

PARTICULAR SOLUTION:

The solution which is free from arbitrary constants is called particular solution.