

### Definite Integral

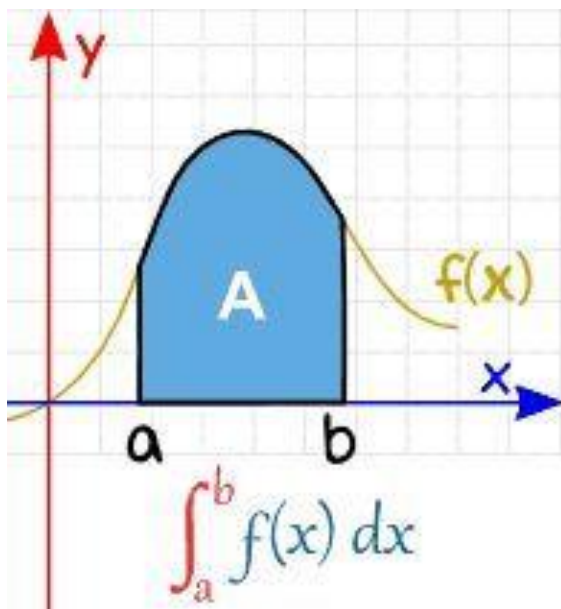
Let  $f(x)$  be a continuous function defined on closed interval  $[a, b]$  then  $\int_a^b f(x)dx$  is called definite integral of  $f(x)$  in the interval  $[a, b]$ .

#### First fundamental theorem of integral calculus

$f(x)$  be a continuous function defined on a closed interval  $[a, b]$  and  $A(x)$  area function)  $A(x) = \int_a^x f(u)du$  for all  $x \in [a, b]$ , then  $\frac{d}{dx} A(x) = f(x)$ . In other words,  $A(x)$  is an anti-derivative of  $f(x)$ .

#### Second fundamental theorem of integral calculus

If  $f(x)$  be a continuous function defined on a closed interval  $[a, b]$  and  $F(x)$  is an anti-derivative of  $f(x)$ , then,  $\int_a^b f(x)dx = F(b) - F(a)$ .



A definite integral is denoted by  $\int_a^b f(x)dx$ , where  $a$  is called the lower limit of the integral and  $b$  is called the upper limit of the integral. The definite integral has a unique value.

$\int_a^b f(x)dx$  defined as the definite integral of  $f(x)$  from  $x = a$  to  $x = b$  denotes area bounded by  $f(x)$   $x$ -axis and  $x = a$  and  $x = b$ .

Note: **There is no need to write integration constant C in definite integration.**

Suppose if we consider  $F(x) + C$  then,

$$\int_a^b f(x) = [F(x) + C]_a^b = (F(b) + C) - (F(a) + C) = F(b) + C - F(a) - C = F(b) - F(a)$$

### **Evaluation of Definite Integrals by Substitution**

Consider a definite integral of the following form

$$I = \int_a^b f[g(x)]g'(x)dx$$

Substitute  $g(x) = t, \Rightarrow g'(x) dx = dt$

When  $x=b, t=g(b)$  and when  $x= a, t = g(a)$

$$\text{Therefore, } I = \int_{g(a)}^{g(b)} f(t)dt$$

Integrate the new integrand with respect to the new variable.