

Class IX Mathematics

Chapter -12

Heron's Formula

Hand out (Module 2/2)

In the previous module we have learnt to find the area of a right angled triangle, isosceles triangle, equilateral triangle and scalene triangle. We also learnt heron's formula to find the area of any triangle where three sides are given.

In this module we shall find the area of quadrilaterals and some plane figures using heron's formula

$$\text{Area of a triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{Where } s = \frac{a+b+c}{2}$$

a, b and c are sides of triangle

Supposed that a farmer has a land to be cultivated and he employs some labourers for this purpose on the term of wages calculated by area cultivated per square metre

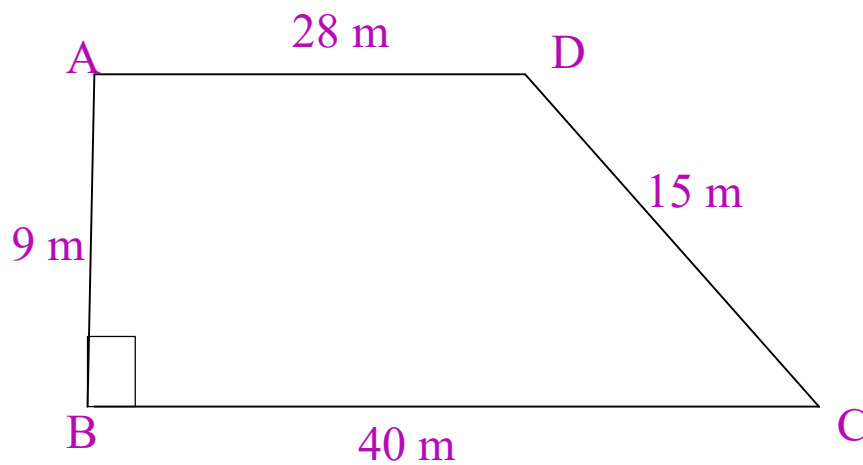
How will you do this?

If a farmer wishes to find the area of his field which is in the shape of a quadrilateral. He needs to divide the quadrilateral in triangular parts and then use the heron's formula to find the area of the triangle

Thus, Heron's formula can be applied to find the area of a quadrilateral by dividing the quadrilateral in two triangular parts also we can use heron's formula to solve the problem from daily life situation.

Ex1. Students of a school staged a rally for cleanliness campaign. They walked through the lanes in two groups, One group walked through the lanes AB, BC and CA while other through AC, CD and DA. Then they cleaned the area enclosed within their lanes. If $AB = 9\text{m}$, $BC = 40\text{ m}$ and $CD = 15\text{m}$, $DA = 28\text{ m}$ and $B = 90$. Which group cleaned more area and by how much? Find the total area cleaned by the students?

Solution :-



In ΔABC

$\angle B = 90$ (By Pythagoras Theorem)

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = 9^2 + 40^2 = 1681$$

$$AC = 41\text{m}$$

$$\begin{aligned} \text{ar}(\Delta ABC) &= \frac{1}{2} \times b \times h \\ &= \frac{1}{2} \times 40 \times 9 \\ &= 180\text{ m}^2 \end{aligned}$$

Now in ΔABC , $AC = 41\text{m}$, $AD = 28\text{m}$, $DC = 15\text{m}$

$$S = \frac{a+b+c}{2} = \frac{28+41+15}{2} = 42\text{m}$$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$A = \sqrt{42(42-28)(42-41)(42-15)}$$

$$A = \sqrt{42 \times 14 \times 1 \times 27}$$

$$A = 126\text{ m}^2$$

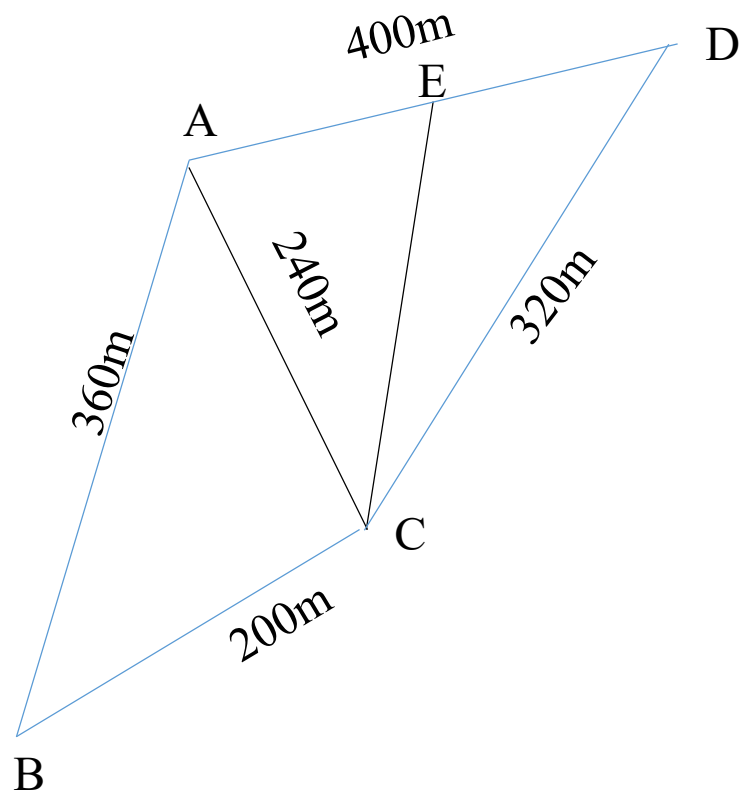
So, first group cleaned 80m^2 and second group cleaned 126m^2

$$\begin{aligned} \text{Total area} &= (180+126)\text{ m}^2 \\ &= 306\text{m}^2 \end{aligned}$$

First group cleaned $180-126 = 56\text{m}^2$ more than the second group.

Ex 2. Kamla has a triangular field with sides 240 m, 200 m and 360 m where she grew wheat. In another triangular field with sides 240 m, 320m, 400m adjacent to the previous field she wanted to grow potatoes and onions. She divided the field in two parts by joining the midpoint of the longest side to the opposite vertex and grew potatoes in one part and onion in the other part. How much area (in hectares) has been used for wheat, potatoes and onions?

Solution :-



Let ΔABC be the field where wheat is grown and ΔACD be the field which has been divided into two parts by joining C to the midpoint of AD

In ΔABD $a = 200\text{m}$

$b = 240\text{m}$

$c = 360\text{m}$

$$S = \frac{a+b+c}{2} = \frac{200+240+360}{2} = 400\text{m}$$

$$A(\Delta ABC) = \sqrt{s(s-a)(s-b)(s-c)}$$

$$A = \sqrt{400(400-200)(400-240)(400-360)}$$

$$A = \sqrt{400 * 200 * 160 * 40}$$

$$A = 16000\sqrt{2} \text{ m}^2 = 1.6\sqrt{2} \text{ hectares} = 2.26 \text{ hectares (Approx.)}$$

Now For ΔACD

$$S = \frac{a+b+c}{2} = \frac{240+400+320}{2} = 480\text{m}$$

$$\text{Area}(\Delta ACD) = \sqrt{s(s-a)(s-b)(s-c)}$$

$$A = \sqrt{480(480-240)(480-400)(480-320)}$$

$$A = \sqrt{480 * 240 * 80 * 160}$$

$$A = 38400 \text{ m}^2 = 3.84 \text{ hectares}$$

Area of $(\Delta AEC) = \text{Area of } (\Delta DEC)$ (same base and same height)

\therefore Area for growing potatoes = area for growing onions

$$= 3.84/2 = 1.92 \text{ hectares}$$

